1 CIRCULAR MOTION

Synopsis

1. **Circular Motion :**

When a body moves such that it keeps equal distance from axis of rotation, the body is in circular motion.

2. **Radius Vector :**

i. Vector drawn from centre to position of body performing circular motion.

ii. Always directed along radius away from centre.

iii. SI units is *metre or m*.

iv. Dimension $[M^0 L^1 T^0]$

v. The position or radius vector of body has constant magnitude but different direction.

3. **Angular Displacement (θ) :**

i. Angle traced by radius vector for a body in circular motion.

ii. Infinite value is scalar, but infinitesimal value is a vector along axis of rotation, given by right hand rule.

iii. SI units is *radian or rad*.

iv. Dimension $[M^0 L^0 T^0]$

4. **Angular Velocity ($\vec{\omega}$) :**

i. Rate of change of angular displacement for a body in circular motion

ii. It is a vector along axis of rotation, given by right hand rule.

iii. SI unit is *rad/sec*.

iv. Dimension $[M^0 L^0 T^{-1}]$.

5. **Angular Acceleration ($\vec{\alpha}$) :**

i. Rate of change of angular velocity for a body is circular motion.

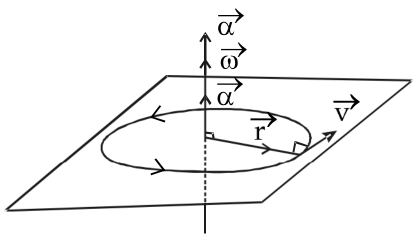
ii. It is a vector along the axis of rotation, given by right hand rule.

iii. a. If 'ω' is increasing, then $\vec{\omega}$ and $\vec{\alpha}$ are in same direction.

b. If 'ω' is decreasing, then $\vec{\omega}$ and $\vec{\alpha}$ are in opposite direction.

iv. SI unit is *rad/s²*

v. Dimension $[M^0 L^0 T^{-2}]$



6. **Relation between linear and angular displacements :**

$$\vec{ds} = \vec{d\theta} \times \vec{r}$$

Where, $\vec{ds}$ = linear displacement

$\vec{d\theta}$ = angular displacement

$\vec{r}$ = radius vector

7. Relation between linear and angular velocities :

$$\vec{v} = \vec{\omega} \times \vec{r}$$

[Note : $\vec{v}$, $\vec{\omega}$ & $\vec{r}$ are all mutually perpendicular to each other.]

8. Relation between linear and angular acceleration :

$$\vec{a} = \left(\vec{\omega}\times\vec{v}\right)+\left(\vec{\alpha}\times\vec{r}\right)$$

$$= \vec{a}_C+\vec{a}_T$$

Magnitude of $\vec{a}$,

$$a = \sqrt{a_C{}^2+a_T{}^2}$$

... (In case of Non U.C.M.)

$$a = a_C$$

... (In case of U.C.M.)

9. Uniform Circular Motion (U.C.M)

i. Circular motion in which *speed* remains constant.

ii. Circular motion in which *magnitude of velocity* remains constant.

iii. Circular motion in which *particle traces* equal angle in equal time interval.

iv. Circular motion in which *angular speed* remains constant.

v. Since direction of tangential velocity changes, U.C.M is an accelerated motion

vi. U.C.M is periodic motion, Time period is given as,

$$T = \frac{2\pi}{\omega}$$

Note : *Time period of Hour Hand* = 43200 sec

Time period of Minute Hand = 3600 sec

Time period of Second Hand = 60 sec

Time period of Earth's rotation = 86400 sec

Also, frequency is given as,

$$f = \frac{\omega}{2\pi} \quad \text{or} \quad \omega = 2\pi f$$

vii. The acceleration existing is the centripetal acceleration, due which one real force, centripetal force acts on the body.

viii. Kinetic energy is constant.

10. Centripetal force :

i. It is always directed *towards* the centre of the circular motion, along the radius.

ii. It is also called as a *real force* as it is produced by some known interaction, example friction, gravitation, electrostatic etc.

iii. Is tano in the *inertial frame*

iv. It is a *necessary condition* for U.C.M.

v. Magnitude is given as or $\frac{mv^2}{r}$ $mr\omega^2$.

11. Centrifugal or Pseudo force :

i. It is always directed *away* from centre of circular motion along the radius.

ii. It is also called as an *imaginary force*.

iii. It arises in the *non intertial frame*.

iv. Applicable in the case of spin drier, centrifuge machine, cream seperator, centrifuge governor etc.

12. Banking of Roads :

i. Angle made by road surface with horizontal is called *Angle of Banking*.

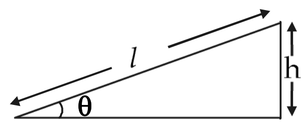
ii. Outer edge of road is always elivated, with respect to inner edge.

iii. Maximum safe velocity along banked road is

$$V_{max} = \sqrt{rg\frac{\left[\mu_s+\tan\theta\right]}{\left[1-\mu_s\tan\theta\right]}}$$

iv. Optimum speed along banked road.

$$V_0 = \sqrt{rg\tan\theta}$$



For small values of θ,

$$V_{max} = \sqrt{rg\frac{h}{l}}$$

iv. Weight of vehicle on a banked road is balanced by the compoment N cos θ and centrifugal force is balanced by the component N sin θ.

13. Bending of Cyclist on a flat Circular road :

i. A cyclist bends to compensate for the centrifugal force.

ii. Maximum safe velocity for the cyclist is

$$V_{max} = \sqrt{rg\tan\theta}$$

where θ = angle made by cyclist with the vetrical.

14. Safe velocity along flat circular road :

i. Along flat circular road, frictional force balances the centrifugal force.

ii. Maximum safe velocity of a vertical is given as,

$$V_{max} = \sqrt{\mu rg}$$

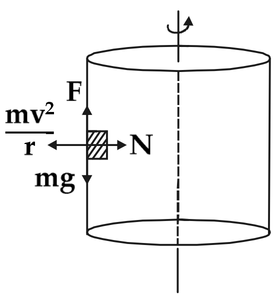
iii. Safe velocity is *independent of mass* of the body in circular motion.

15. Concept of a Rotor :

i. A body performing horizontal circular motion along a vertical drum.

ii. Also can be compared to a motorcylist performing horizontal circular motion in a *death well*.

iii. Centrifugal force is balance by the normal reaction.



$$\frac{mv^2}{r} = N$$

iv. Friction force is balanced by the weight.

$$mg = F$$

v. For body to remain pinned against wall of vertical drum

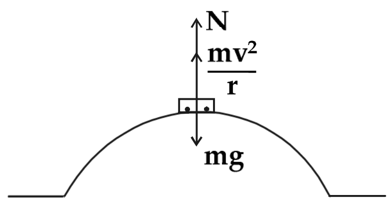
$$\mu N = mg$$

$$\omega = \sqrt{\frac{g}{\mu r}}$$

$$v = \sqrt{\frac{rg}{\mu}}$$

16. Concept of a Convex bridge :

i. A vehicle moving along a convex bridge has, normal reaction given as,



$$N = mg - \frac{mv^2}{r}$$

ii. A vehicle moving with maximum speed along convex bridge has normal reaction zero, i.e. N = 0,

$$\therefore \frac{mv^2}{r} = mg$$

$$V_{(max)} = \sqrt{rg}$$

17. Concept of Conical Pendulum :

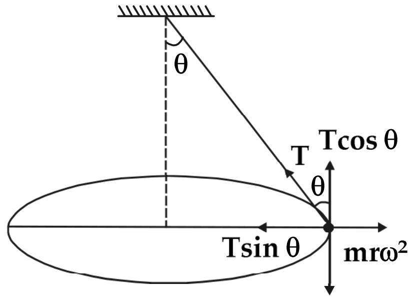
Consider a body of mass m is revolving in a horizontal circle of radius r.

Tension (T) resolved into two mutually perpendicular components.

i. T cos θ which balances weight

$\therefore$ T cos θ = mg

ii. T sin θ which provides necessary centripetal force



$\therefore \quad T\sin\theta = mr\omega^2$

$\therefore \quad \tan\theta = \frac{r\omega^2}{g}$

$\omega^2 = \frac{g\tan\theta}{r}$

i.e. $\omega = \sqrt{\frac{g\tan\theta}{r}}$

$\therefore \quad \frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{r}}$

$T = 2\pi\sqrt{\frac{l\sin\theta}{g\tan\theta}}$

$= 2\pi\sqrt{\frac{l\sin\theta}{g\sin\theta/\cos\theta}}$

(Where $r = l\sin\theta$)

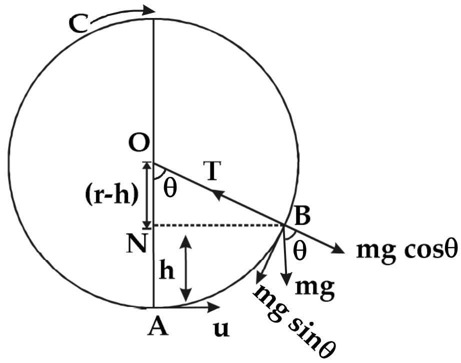
$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$= 2\pi\sqrt{\frac{h}{g}}$

18. Concept of Vertical Circular Motion :

Let us consider a particle (say stone) of mass 'm' tied to a string of length 'r' in motion from the lowest point A in anti-clockwise direction with an velocity 'u' rising at height 'h'.

Let at any instant 't' the particle reaches point B decribing angle θ from its original position.



$\therefore \quad T - mg\cos\theta = \frac{mv^2}{r} \quad ...\text{(i)}$

where v = velocity of the particle at point B.

If the effective vertical height through which particle has risen, while moving from A to B be h (i.e., AN = h), then we have

$v^2 = u^2 - 2gh \quad ...\text{(ii)}$

Substituting value of v^2 in (i), we have

$T - mg\cos\theta = \frac{m}{r}\left(u^2 - 2gh\right)$

or $\quad T = mg\cos\theta + \frac{m}{r}\left(u^2 - 2gh\right)$

But from ΔBON,

$\cos\theta = \frac{ON}{OB} = \frac{OA - AN}{OB}$

$= \frac{r-h}{r}$

$T = mg\left(\frac{r-h}{r}\right) + \frac{m}{r}\left(u^2 - 2gh\right)$

$= \frac{m}{r}[gr - gh + u^2 - 2gh]$

or $\quad T = \frac{m}{r}[u^2 + gr - 3gh] \quad ...\text{(iii)}$

This is the general equation of motion in a vertical circle giving us value of tension present in the string at any point during its motion.

Special cases :

i. At the lowermost (or starting) point A, h = 0 and hence tension at the lowest point :

$$T_L = \frac{m}{r}\left[u^2 + gr\right] \quad \text{... (iv)}$$

ii. At the uppermost point C, h = 2r and hence tension at the point is given by

$$T_H = \frac{m}{r}\left[u^2 + gr - 6gr\right]$$

$$\text{or } T_H = \frac{m}{r}\left[u^2 - 5gr\right] \qquad \text{... (v)}$$

Subtracting (iv) from (v), we have

$$T_L - T_H = \frac{m}{r}\left[u^2 + gr\right] - \frac{m}{r}\left[u^2 - 5gr\right]$$

$$\therefore\ T_L - T_H = \mathbf{6mg} \qquad \text{... (vi)}$$

Thus, difference in tension at the lowest and highest point is equal to six times the weight of the revolving particle

iii. If the initial velocity u is just sufficient to make the stone cross the highest point C without any slackening of string, then $T_H = 0$ and hence

$$0 = \frac{m}{r}\left[u^2 - 5gr\right]$$

$$\text{or } u^2 = 5gr \qquad \text{[From equation (v)]}$$

$$\text{or } u = \sqrt{5gr}$$

Hence, the least velocity with which a particle (say stone) must be projected from the lowest position of a vertical circle so as to reach the highest point without leaving the track is $\sqrt{5gr}$. This is the minimum velocity with which the stone can LOOP the LOOP.

iv. Moreover, if V is the velocity which the stone possesses at highest point C in case of no slackening of string, then

$$V^2 = u^2 - 2g.\,2r \qquad [\because \text{At highest point } h = 2r]$$

$$= u^2 - 4gr$$

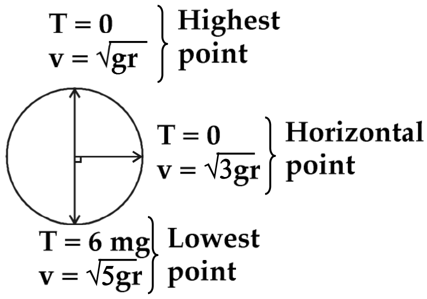
$$\text{or } V = \sqrt{gr}$$

Velocity $V = \sqrt{gr}$ is commonly called as the *'Critical Velocity'*.

Some common example of motion in a vertical plane are an aeroplane looping a loop, motor-cyclist looping the loop in a "globe of death" in circus or motion of a car on a circular bridge.

Tension (T)

At any point 'p'



$$T = \frac{mv^2}{r} + mg\cos\theta$$

At lowest point

$$T = \frac{mv^2}{r} + mg$$

At highest point

$$T = \frac{mv^2}{r} + mg$$

At a point along horizontal position

$$T = \frac{mv^2}{r}$$

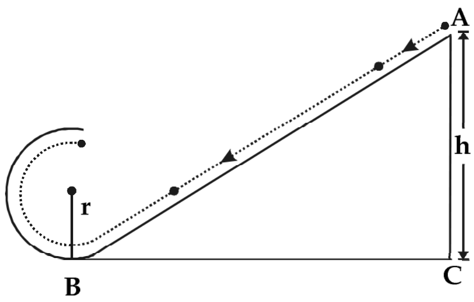
Position	**Velocities (V)**	**Acceleration**
Lowest Point	$V \geq \sqrt{5rg}$	$a \geq \sqrt{3g}$
Highest Point	$V \geq \sqrt{rg}$	$a \geq \sqrt{g}$
Horizontal Point	$V \geq \sqrt{3rg}$	$a \geq \sqrt{5g}$

Energy

At lowest point	At highest point	At horizontal point
K.E. $\geq \frac{1}{2}mv^2$	K.E. $\geq \frac{1}{2}mv^2$	K.E. $\geq \frac{1}{2}mv^2$
$\geq \frac{1}{2}m(5\ rg)$	$\geq \frac{1}{2}mgr$	$\geq \frac{1}{2}m\ 3rg$
$\geq \frac{5}{3}mrg$	P.E. = mgh	$\geq \frac{3}{2}mgr$
P.E. = 0		= mgh (h = r)

19. Height of inclined plane for looping the loop of circle of radius 'r'.

If v is the velocity at the bottom of the incline of height 'h' then according to the principle of conservation of energy.



P.E. at the top of the incline = K.E. at the bottom of the incline

i.e. $$mgh = \frac{1}{2}mv^2$$

or $$h = \frac{v^2}{2g} \quad \dots \text{(i)}$$

Further, the motor-cyclist will be able to go around the loop if the velocity at the lowest point of the loop is at least $\sqrt{5gr}$

i.e. $$v = \sqrt{5gr}$$

$$v^2 = 5gr \quad \dots \text{(ii)}$$

$\therefore$ From (i) and (ii)

$$h = \frac{5gr}{2g} = \frac{5r}{2}$$

20. Kinematic equations for accelerated circular motion :

$$\omega_2 = \omega_1 + \alpha t$$

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$\theta_p = \omega_1 + \frac{1}{2}\alpha(2p - 1)$$

ω_1 = initial angular velocity

α = angular acceleration

ω_2 = final angular velocity

θ = total angular displacement

θ_p = angular displacement in particular pth second

CLASSWORK

Multiple Choice Questions

1.1. ANGULAR DISPLACEMENT :

(1) A particle is moving in a uniform circular motion with radius 'r' . In quarter revolution, the distance and displacement covered by the particle is....

(a) πr, r (b) 0.5 πr, $\sqrt{2r}$

(c) 0.5 πr, 2r (d) πr, $\sqrt{2r}$

(2) Which of the following is NOT a correct statement about angular displacement?

(a) It is dimensionless quantity.

(b) In vectors, $\overrightarrow{\delta s} = \overrightarrow{\delta \theta} \times \overrightarrow{r}$

(c) Direction of angular dispalcement is perpendicular to plane and directed upwards if particle describes its motion in anticlockwise direction

(d) The instantaneous angular displacement and radius vectors are mutually parallel to each other.

(3) The angular displacement of the minute hand of clock in 20 minutes is :

(a) $\frac{\pi}{2}$ radian (b) $\frac{2\pi}{3}$ radian

(c) $\frac{4\pi}{3}$ radian (d) 360^0

(4) An athelete runs on a circular track of radius 25 m with distance 400 m. The angle traced by radius vector at the axis of circular path is....

(a) 10^c (b) 16^c (c) 24^c (d) 55^c

(5) The direction of angular displacement in U.C.M. is given by

(a) left hand rule

(b) right hand thumb rule

(c) right handed screw rule

(d) either 'b' or 'c'

1.2. ANGULAR VELOCITY AND ANGULAR ACCELERATION:

(6) The ratio of the angular speeds of the hour and the minute hand of a clock is

(a) 1 : 12 (b) 1 : 6 (c) 1 : 8 (d) 12 : 1

(7) A body moving in a circle at constant speed has an acceleration which is constant in,

(a) magnitude only

(b) direction only

(c) both magnitude and direction

(d) none of these

(8) A particle moves along a circular path of radius 20 cm with a constant angular acceleration of 4 rad/s² If the initial angular speed of the particle is 2 rad/s, then the angular displacement of the particle after 5 second will be

(a) 30 radian (b) 40 radian

(c) 50 radian (d) 60 radian

(9) A particle moves in a circle of radius 25 cm at 2 r.p.s. The acceleration of the particle in metre per sec² is

(a) $12\pi^2$ (b) $8\pi^2$ (c) $4\pi^2$ (d) 2π

(10) A coin is placed on a rotating turntable just slips, if it placed at a distance of 4cm from the centre. If the angular velocity of the turn table isdoubled, it will just slip at a distance of

(a) 1 cm (b) 3 cm (c) 4 cm (d) 5 cm

(11) The angular displacement of a particle performing UCM is given by $\theta = 2t^3 - \frac{t^2}{4} + 4t$ where θ is in radian. At the end of 1.5 s, the angular acceleration will be

(a) 16 rad /s² (b) 17.5 rad/s²

(c) 18 rad /s² (d) 22.5 rad/s²

(12) An object is moving in a circle of radius 100 m with a constant speed of 31.4 m/s. What is its average speed for one complete rotation?

(a) zero (b) 31.4m/s

(c) 3.14 m/s (d) $\sqrt{2} \times 31.4$ m/s

(13) A particle moves along a circular orbit with constant velocity. This necessarily means.

(a) its motion is confined to a single plane

(b) its motion is not cofined to a single plane

(c) nothing can be said regarding the plane motion

(d) its motion is one-dimensional

(14) A body is moving in a circular path with a constant speed. It has

(a) magnitude only

(b) direction only

(c) both magnitude and direction

(d) none of these

(15) An hour hand of a watch is 2.5 cm long. The linear speed of a point on hour hand at a distance of 0.5 cm from the tip is......

(a) 2×10^{-4} m/s (b) 2.9×10^{-6} m/s

(c) 3×10^{-5} m/s (d) 4×10^{-7} m/s

1.3. RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY :

(16) Angular velocity is related to the equivalent linear velocity by the relation

(a) $v = \frac{\omega}{r}$ (b) $v = \frac{r}{\omega}$

(c) $\boldsymbol{\omega} = \frac{v}{r}$ (d) $\boldsymbol{\omega} = \frac{r}{v}$

(17) The linear velocity of a point on the equator of earth of radius 6400 km is nearly

(a) 450 m/s (b) 466 m/s

(c) 480 m/s (d) 539 m/s

(18) The angular velocity of a wheel is 70 rad/sec. If the radius of the wheel is 0.5 m, then liner velocity of the wheel is:

(a) 35 m/s (b) 20m/s

(c) 70 m/s (c) 10 m/s

(19) If the length of the seconds hand in a top clock is 3 cm, the angular velocity and linear velocity of the tip is

(a) 0.2047 rad/sec ; 0.0314 m/sec

(b) 0.2547 rad/sec ; 0.314 m/sec

(c) 0.1472 rad/sec ; 0.06314 m/sec

(d) 0.1047 rad/sec ; 0.00314 m/sec

(20) A particle starts from rest moves with an angular acceleration of 3 rad/s² in a circle of redius 3m. Its liner speed after 5 second will be

(a) 15 m/s (b) 80**p**

(c) 45 m/s (d) 7.5 m/s

(21) A wheel of radius 0.5m make 60 revolutions per minute. The liner speed (in m/s) of a point on its circumference is:

(a) $\pi/72$ (b) π (c) 3π (d) 6π

1.4 UNIFORM CIRCULAR MOTION (U.C.M) :

(22) A particle is moving in a horizontal circle with constant speed. It has constant

(a) Velocity (b) Acceleration

(c) Kinetic energy (d) Displacement

(23) In uniform circular motion

(a) both velocity and acceleration are constant

(b) both velocity acceleration change

(c) acceleration and speed are constant but velocity changes

(d) acceleration and speed both change

(24) A particle is moving along a circular path of radius 7 m with uniform speed of 7 m/s. The time taken by particle for one and a half revolution will be

(a) 3.14 s (b) 6.28 s (c) 9.42 s (d) 10.5 s

(25) In a uniform circular motion,

(a) work done is zero

(b) torque is zero

(c) angular speed constant

(d) all of the above

1.5. ACCELERATION IN UNIFORM CIRCULAR MOTION (RADIAL ACCELERATION) :

(26) A body performing nonuniform circular motion experiences linear acceleration a_r and tangential acceleration a_T, such that a_r changes the direction of linear velocity, while a_T changes the magnitude of linear velocity. Then the resultant acceleration 'a' in the case is,

(a) $\sqrt{a_T^3 + a_T^3}$ (b) $a = a_r + a_T$

(c) $a = \sqrt{a_T^2 + a_T^2}$ (d) $a = \sqrt{a_r^2 a_T^2}$

(27) What happens to the centripetal acceleration of a revolving body if you double the arbital speed V and halve the angular speed ω ?

(a) the centripetal acceleration remains unchaned

(b) the centripetal acceleration is halved

(c) the centripetal acceleration is doubled

(d) the centripetal acceleration is quadrupled

(28) A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' m/s^2, then the resultant acceleration will be ...

(a) $\sqrt{\left(\frac{v^2}{r^2} - a^2\right)}$ (b) $\sqrt{\left(\frac{v^4}{r^2} + a^2\right)}$

(c) $\sqrt{\left(\frac{v^2}{r^2} + a^2\right)}$ (d) $\sqrt{\left(\frac{v^4}{r^2} - a^2\right)}$

(29) The angular speed of a particle, moving in a circular of radius 20cm, increase from 2 rad/s to 40 rad/s in 19 s the ratio of its centripetal acceleration to tangential acceleration at the end of 19 s is,

(a) 400 : 1 (b) 1 : 800 (c) 1 : 400 (d) 800 : 1

(30) An object moves at a constant speed along a circular path in a horizontal XY plane with the centre at the origin. When the object is at x = 2m, its velocity is $-(4\ m/s)\hat{j}$. What is the objects acceleration when it is at y = 2m.

(a) $-\left(8m/s^2\right)\hat{j}$ (b) $-\left(8m/s^2\right)\hat{i}$

(c) $-\left(4m/s^2\right)\hat{j}$ (d) $4m/s^2)\hat{i}$

1.6. CENTRIPETAL AND CENTRIFUGAL FORCES :

(31) A body is moving with a uniform speed along a circle. If its direction of motion is reversed but speed is kept constant then

(a) the centrifugal force will suffer change in direction in word

(b) the centripetal force will not suffer any change in direction

(c) the centripetal force will have its direction reversed

(d) both 'a' and 'b'

(32) A string breaks if its tension exceeds 10 newtons.

A stone of mass 250 gm tied string of length 10 cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be

(a) 20 rad/s (b) 40 rad/s

(c) 100 rad/s (d) 200 rad/s

(33) Four point masses, each of 1 kg are joined together by string which forms a square of diagonal 0.707 m. If the square is placed on a rotating table which is rotated with a frequency of 5 rps, then the tension in the string will be ...

(a) 24.68 N (b) 246.8 N

(c) 2.468 N (d) 0.2468 N

(34) When a bucket of water is whirled in a vertical circle fast enough water does not fall from bucket in its highest position because,

(a) the centrifugal force is less than the weight of water

(b) the centrifugal force is more than theweight of water

(c) water at the highest position of the bucket loses all weight

(d) none of these

(35) If the object is moving in circular path at constant speed of 4m/s. calculate centripetal force required to hold 2 kg object by 1m long string

(a) 16 N (b) 8 N (c) 4 N (c) 32 N

1.7. BANKING OF ROADS :

(36) A train is moving with a speed v on a curved railway track of radius r. A spring balance loaded with a block of mass m is suspended from the roof of the train. The reading of spring balance is ...

(a) mg (b) $\frac{mv^2}{r}$

(c) $mg+\left(\frac{mv^2}{r}\right)$ (d) $\sqrt{(mg)^2+\left(\frac{mv^2}{r}\right)^2}$

(37) While taking a turn on a curved road, a cyclist has bend through a certain angle. This is done

(a) to reduce his speed

(b) to decrease the friction between the tyres and the road

(c) to get the necessary centripetal force

(d) to reduce his weight

(38) A road is 10 m wide. Its radius of curvature is 50 m . The outer edge is above the lower edge by a distance of 1.5 m. This road is most suited for a velocity.

(a) 2.5m/sec (b) 4.5 m/sec

(c) 6.5m/sec (d) 8.5m./sec

(39) Keeping the banking angle same, to increase the maximum speed with which a vehicle can travel on the curve road by 10%, the radius of curvature of the road has to be changed from 20m to

(a) 16 m (b) 18 m (c) 24.2 m (d) 30.5 m

(40) An automobile is turning around a circular road of radius r. The coefficients of friction between the tyres and the road is μ. For safety of the vehicle, its velocity should not be more than

(a) $\frac{\sqrt{\mu g}}{r}$ (b) $\sqrt{\mu g r}$ (c) $\mu g r$ (d) $\frac{\mu g}{r}$

(41) Banking of roads at curve is necessary so as to avoid, ...

(1) The dependence of centripetal force on force of friction.

(2) Overturning of vehicle moving with maximum speed

(3) rough nature of road surface which increases the force of friction and causes the wear and tear of tyres of vehicle.

(4) skidding of the vehicle.

The correct statement is/are

(a) 1, 2 (b) 1, 2, 3 (c) 1, 3 (d) 1, 2, 4

(42) A car is moving in a horizontal circular track of radius 10m with a constant speed 10 m/s. If a bob is suspended from the roof of the car by a light string, it performs horizontal circular motion. Its time period will be ....

(a) 1.57 s (b) 3.14 s (c) 6.28 s (d) 9.1 s

(43) What is the angle of banking of a railway track of radius of curvature 250 m, if the maximum velocity of th train is 90 km/hr.?

(use $g = 10\ m/s^2$)

(a) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ (b) $\theta = \tan^{-1}\left(\frac{1}{3}\right)$

(c) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (d) $\theta = \tan^{-1}\left(\frac{1}{5}\right)$

(44) When a body is kept on a rough disc rotating in a horizontal plane about an axis perpendicular to its plane and passing through its centre, the centripetal force is provided by frictional force between the surface of the body and the disc. When the body is about to fly off the disc, we have

(a) $\frac{\mu - v^2}{R} = mg$ (b) $\tan\theta = \frac{v^2}{Rg}$

(c) $mr\omega^2 = \mu mg$ (d) none of these

(45) In a conical pendulum, the centripetal force $\left(\frac{mv}{r}\right)^2$ acting on the bob is given by

(a) $\frac{mgr}{\sqrt{L^2 - r^2}}$ (b) $\frac{mgr}{L^2 - r^2}$

(c) $\frac{\left(L^2 - r^2\right)^{1/2}}{mgL}$ (d) $\frac{mgL}{\left(L^2 - r^2\right)^{1/2}}$

(46) A motor cyclist moving with a velocity of 72 km per hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 metres. The acceleration due to gravity is 10 m/sec². In order to avoid skidding he must not bend with respect to the vertical plane by an angle greater than

(a) $\theta = \tan^{-1}(6)$ (b) $\theta = \tan^{-1}(25.92)$

(c) $\theta = \tan^{-1}(2)$ (d) $\theta = \tan^{-1}(4)$

(47) A coin kept on a horizontal rotation disc has its centre at a distance of 0.25 m from the axis of rotation of the disc. If μ_s is 0.2, then the angular velocity of the disc at which the coin will just slip off ($g = 9.8\ m/s^2$) is ...

(a) 3.8 rad/s (b) 2.8 rad/s

(c) 4.8 rad/s (d) 5.8 rad/s

(48) Consider a simple pendulum of length 1m. Its bob performs a circular motion in horizontal plane with its string making an angle 60^0 with the vertical. The period of rotation of the bob is (Take $g = 10 m/s^2$)

(a) 2 s (b) 1.4 s

(c) 1.98 s (d) 2.4 s

(49) The length of the string of a conical pendulum is 10 m and it has bob of mass 50 g. The angle that the string makes with the vertical is 300. If the bob covers one revolution in 3 s, then the corresponding centripetal force acting on the bob will be

(a) 10 N (b) 1 N

(c) 100 N (d) 5 N

(50) A bob having a diameter of 3 cm with mass 100g is joined at the end of the string having length 48.5 cm. If the bob is rotated at 600 r.p.m, thenthe tension in string is (neglect the weigth and string of the bob).

(a) 150.9N (b) 100 N

(c) 197. N (d) 297.9 N

(51) A simple pendulum having a length of 2m and mass of the bob 200g. When the tension in the string becomes more than 4 N, it breaks what is the maximum angle through which the string makes with the vertical if the bob is whirled in a horizontal plane (take $g = 10\ ms^2$)

(a) 0^0 (b) 30^0

(c) 45^0 (d) 60^0

1.8. VERTICAL CIRCULAR MOTION DUE TO EARTH'S GRAVITATION

(52) When a bucket of water is whirled in a vertical circle fast enough water does not fall from bucket in its highest position because,

(a) the centrifugal force is less than the weight of water

(b) the centrifugal force is more than the weight of water

(c) water at the highest position of the bucket loses all weight

(d) none of these

(53) A particle of mass m is tied to one end of a string and revolved in a vertical circle. In its motion there are only two points where the tension in the string is equal in magnitude. They are

(a) at the ends of the vertical diameter

(b) at the ends of th e horizontal diameter

(c) the centre of the circle and the end of the diameter

(d) at the end of any diameter inclined at an angle θ to the horizontal, where $\theta > O^0$

(54) A small sphere is attached to a string and rotated in a vertical circle about its other end. The speed of the sphere is slowly increased. The string is likely to break at the orientation when the sphere is at

(a) the lowest point

(b) the highest point

(c) when it is at the ends of the horizontal diameter

(d) none of the above

(55) A vertical circular motion is a non uniform circular motion because ...

(a) acceleration due to gravity affects the motion of the body.

(b) kinetic energy of body changes at every point.

(c) speed of body changes at every point.

(d) All of above.

1.9. EQUATION FOR VELOCITY AND ENERGY AT DIFFERENT POSITIONS IN VERTICAL CIRCULAR MOTION:

(56) A sphere is suspended by a thread of length l. What minimum horizontal velocity should be imparted to the sphere, so that it will reach the height of suspension ?

(a) $\sqrt{gl}$ (b) gl (c) $2gl$ (d) $\sqrt{2gl}$

(57) A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed v. When the string becomes horizontal, the magnitude of the change in velocity is given by

(a) $\sqrt{2gL}$ (b) $\sqrt{v^2 - 2gL}$

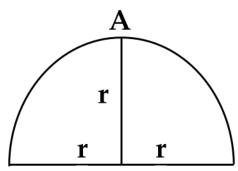
(c) $\sqrt{v^2 - gL}$ (d) $\sqrt{2\left(v^2 - gL\right)}$

(58) A sphere of mass 0.2 kg is attached to an inextensible string of length 0.5 m whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 0.3 m. The speed of the sphere will be

(a) $1.5\ ms^{-1}$ (b) $2.5\ ms^{-1}$

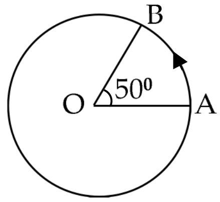
(c) $3.2\ ms^{-1}$ (d) $4.7\ ms^{-1}$

(59) A body of mass m slides from rest, down the surface of a smooth hemispherical bowl of radius r from the highest point A. What is the velocity of the body when if reaches the bottom?



(a) $\sqrt{gr}$ (b) $\sqrt{2gr}$ (c) $\sqrt{3gr}$ (d) 2 mgr

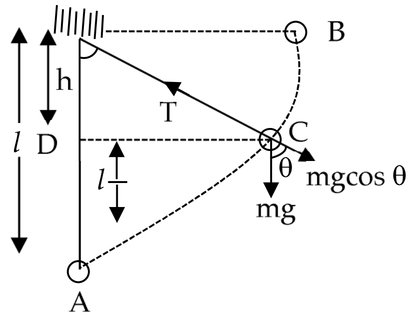
(60) A particle is moving in a circle of radius r with constant speed $\mathbf{v}$. The change in velocity of particle while moving from A to B ($\angle AOB = 50^0$) is



(a) $2\ \mathbf{v} \cos 50^0$ (b) $2\ \mathbf{v} \sin 50^0$

(c) $2\ \mathbf{v} \cos 25^0$ (d) $2\ \mathbf{v} \sin 25^0$

(61) A simple pendulum of mass m swings with an angular amplitude of 60^0 when its angular displacement is 30^0. The tension in the string is



(a) $\frac{1}{2}mg\left(3\sqrt{3}-2\right)N$ (b) $mg\left(2\sqrt{3}-2\right)N$

(c) $mg\left(3\sqrt{3}-2\right)N$ (d) $2\,mg\left(3\sqrt{3}-2\right)N$

(62) A body of mass m rotated along a verticle circle with the help of a light string such that velocity of the body at any point is critical. If T_1 and T_2 are tensions in the string when the body is crossing the highest and lowest points of the vertical circle respectively, then

(a) $T_2 - T_1 = 6$ mg (b) $T_2 - T_1 = 4$ mg

(c) $T_2 - T_1 = 3$ mg (d) $T_2 - T_1 = 2$ mg

(63) A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angle 50^0 and 60^0 from vertical (lowest position) are T_1 and T_2, respectively, then

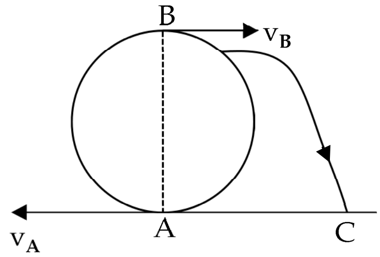
(a) $T_1 = T_2$

(b) $T_2 > T_1$

(c) $T_1 > T_2$

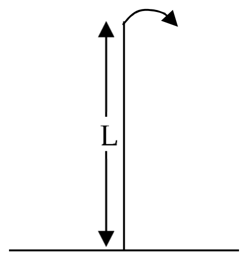
(d) tension in string always remains the same.

(64) An object is tied to a string of length l and is revolved in a vertical circle with the minimum velocity. When the object reaches the upper most point, the string breaks and it describes a parabolic path as shown in the figure under the gravitational force. The horizontal range AC in the plane of A would be ...



(a) l (b) $2l$ (c) $\sqrt{2}l$ (d) $2\sqrt{2}l$

(65) A thin uniform rod of mass M and length L is positioned vertically above an anchored frictionless pivot point, as shown in figure and then allowed to fall to ground. With what speed does the free end of the rod strike the ground?



(a) $\sqrt{gL}$ (b) $\sqrt{2gL}$ (c) $\sqrt{5gL}$ (d) $\sqrt{3gL}$

(66) A stone weighing 2 kg is whirled in a vertical circle attached to the end of a rope of length 1m. The tensions at lowest, midway and highest positions are respectively ... (Take g = $10 m/s^2$)

(a) 0 N, 60 N, 120 N (b) 120 N, 60 N, 0 N

(c) 60 N, 120 N, 0 N (d) 0 N, 120 N, 60 N

1.10. KINEMATICAL EQUATIONS FOR CIRCULAR MOTION IN ANALOGY WITH LINER MOTION :

(67) A table fan attains speed of 120 rpm in 10s. How many rotations have the blades performed till reaching this speed

(a) 5 (b) 10 (c) 12 (d) 20

(68) A wheel which is initially at rest is subjected to an angular acceleration and it completes 10 rotations in time 't'. Then the time taken by it to complete the next 10 rotations is

(a) 2t (b) $\sqrt{2}$ t

(c) $\left(\sqrt{2}-1\right)$t (d) $\left(\sqrt{2}+1\right)$t

(69) The frequency of a particle performing circular motion changes from 60 rpm to 180 rpm in 30 s. The angular displacement at 20 s from start is ...

(a) 55 rad (b) 101.4 rad

(c) 209 rad (d) 300 rad

(70) The kinematical equations of motion are applied to solve the problems of circular motion, because of

(a) the acceleration is non uniform

(b) the acceleration is uniform

(c) the acceleration and velocity are uniform

(d) the motion is circular

(69) The shaft of a motor car rotates at constant angular frequency of 3000 revolutions/min. The angle through which it has turned in one second in radians is

(a) 100 π (b) 50 π (c) 25 π (d) 125 π

(70) Initial angular velocity of a wheel is 2 rad/s. It rotates with a constant angular acceleration of 3.5 rad/s^2. Its angular displacement in 2 s is

(a) 4 rad (b) 7 rad (c) 8 rad (d) 11 rad

HOME WORK

Multiple Choice Questions

1.1. ANGULAR DISPLACEMENT :

(1) If a particle completes half revolution along the circumference of a circle then its angular displacement is .....

(a) 0 (b) π (c) 2π (d) 3π

(2) The angular displacement of a second hand of a clock in 15 s in SI unit is

(a) π radian (b) 180^0

(c) 90^0 (d) $\frac{\pi}{2}$ radian

(3) The infinitesimal angular displacement of a particle performing uniform circular motion is a vector because .....

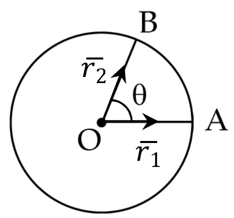
(a) it obeys the cumulative and associative laws of vector addition.

(b) it do not obey the laws of vector addition.

(c) it do not obey the laws of multiplication of vectors.

(d) it changes with time.

(4) In figure below, the change in the magnitude of position vector (or the displacement) $\Delta \vec{r}$ of the particle from position A to position B is ......



(a) $2r \sin \frac{\theta}{2}$ (b) $r \sin \frac{\theta}{2}$

(c) $r \cos \frac{\theta}{2}$ (d) $2r \cos \frac{\theta}{2}$

(5) In half revolution, the difference between linear distance and displacement of particle is .....

(a) π r (b) 1.14 r (c) r (d) 0

(6) Finite angular displacement is not a vector because .....

(a) it does not obey the law of vector addition.

(b) it obeys the law of vector addtion.

(c) its direction is given by right hand rule.

(d) it changes with time.

(7) The angular displacement of the minute hand in 15 minutes is ......

(a) 1.57 rad (b) π rad

(c) 2.9 rad (d) 3.2 rad

(8) A wheel rotates with a constant angular velocity of 300 r.p.m. the angle through which the wheel rotates in one second is :

(a) π (b) 5 π rad

(c) 10 π rad (d) 20 π rad

1.2. ANGULAR VELOCITY AND ANGULAR ACCELERATION:

(9) The ratio of angular speeds of minute-hand and hour-hand of a watch is :

(a) 1 : 12 (b) 6 : 1 (c) 12 : 1 (d) 1 : 6

(10) The angular speed of a motor increases from 200 rpm to 400 rpm in 20 ms. The angular acceleration of the motor is ....

(a) 1.047 rad/s² (b) 1.047×10^3 rad/s²

(c) 2.05 rad/s² (d) 2.05×10^3 rad/s²

(11) The angular velocity is perpendicular to the plane of rotation and ....

(a) directed upwards for clockwise direction and downwards for anticlockwise direction.

(b) directed upwards for anticlockwise direction and downwards for clockwise direction

(c) directed upwards for both clockwise and anticlockwise directions.

(d) directed downwards for both clockwise and anticlockwise directions.

(12) An object of mass 100 grams is whirled in a horizontal circle of radius 1 meter. If it performs 120 revolutions per minute, its angular velocity is

(a) 4π rad/s (b) 2π rad/s

(c) π rad/s (d) $\pi/2$ rad/s

(13) An electric motor operates at 1200 rpm. Its angular velocity will be

(a) 40π rad s^{-1} (b) 20π rad s^{-1}

(c) 30π rad s^{-1} (d) 25 rad s^{-1}

(14) The ratio of angular velocity between second hand, minute hand and hour hand of a clock in rad/min. is ......

(a) 1 : 2 : 5 (b) 1 : 2 : 3

(c) 6 : 3 : 1 (d) 720 : 12 : 1

(15) The angular displacement of a particle performing circular motion is $\theta = \frac{t^3}{60} - \frac{t}{4}$ where θ is in radian and 't' is in seconds. Then the angular velocity and angular acceleration of a particle at the end of 5 s will be

(a) 1 rad/s, 5 rad/s^2

(b) 1 rad/s, 0.5 rad/s^2

(c) 5 rad/s, 1 rad/s^2

(d) 0.1 rad/s, 5 rad/s^2

(16) The angular dispalcement θ of a flywheel varies with time as $\theta = at + bt^2 + ct^3$ then its angular acceleration is given by

(a) a + 2b + 3c (b) 2b + 6ct

(c) 2b – 8ct (d) 2b + 12ct

(17) If a particle is describing circular path of radius 10 m nevery 2 s, then the average angular speed of the particle during 4 s will be

(a) 0.5π rad/s (b) $3\pi/4$ rad/s

(c) 20π rad/s (d) π rad/s

(18) A particle moves along a circle of radius 10 cm. If its linear speed changes from 4 m/s to 5 m/s in 1 s, then its angular acceleration will be

(a) 2 rad/s^2 (b) 5 rad/s^2

(c) 10 rad/s^2 (d) π rad/s^2

(19) The angular speed of a flywheel rotating at 90 r.p.m. is

(a) π rad/s (b) 2π rad/s

(c) 4π rad/s (d) 3π rad/s

(20) The equation for the angular displacement of a particle moving along a circular path is given by θ+ $2t^3$ + 0.5 where θ is in radians and t is in seconds. The angular velocity of the particle at time t = 2 second is

(a) 12 radian/sec (b) 18 radian/sec

(c) 24 radian/sec (d) 30 radian/sec

(21) A particle P is moving in a circle of radius r with a uniform speed v. C is the centre of the circle, and AB is a diameter. If P is at B, the angular velocities of P about A and C are in the ratio

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 4 : 1

(22) The rate of change of angular displacement in uniform circular motion is

(a) angular velocity $(\vec{\omega})$

(b) angular speed (ω)

(c) angular acceleration $(\vec{\alpha})$

(d) radial acceleration

(23) The SI unit of angular acceleraton is,

(a) radian /s (b) radian /s

(c) radian /s^2 (d) meter /s^2

(24) The angular acceleration of a particle moving along a circular path with uniform speed is

(a) zero (b) variable

(c) infinite (d) cannot be determined

(25) Two particles A and B move in concentric circles of radii r_1 and r_2 respectively in such a way that A, B and the centre of the circel (O) always liie on a straihgt line. The ratio of their angular velocities $\left(\frac{\omega_1}{\omega_2}\right)$ is

(a) $\frac{r_1}{r_2}$ (b) $\frac{r_2}{r_1}$ (c) one (d) $\frac{1}{2}$

1.3. RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY :

(26) An hour hand of watch is 2.5 cm long. The linear speed of a point on hour hand at a distance of 0.5 cm from the tip is ....

(a) 2×10^{-4} m/s (b) 2.9×10^{-6} m/s

(c) 3×10^{-5} m/s (d) 4×10^{-7} m/s

(27) A small steel sphere tied at the end of a string is whirled in a horizontal circle with uniform angular velocity ω_1. The string is suddenly pulled so that the radius of the circle is halved. If ω_2 is the new angular velocity then

(a) $\omega_1 = \omega_2$ (b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$ (d) $\omega_1 = 2\omega_2$

(28) The vector equation connecting the position vector $\vec{r}$, angular velocity $\vec{\omega}$ and tangential velocity $\vec{v}$ is given by

(a) $\vec{v} = \vec{\omega} \cdot \vec{r}$ (b) $\vec{v} = \dfrac{\vec{\omega}}{\vec{r}}$

(c) $\vec{v} = \dfrac{\vec{r}}{\vec{\omega}}$ (d) $\vec{v} = \vec{\omega} \times \vec{r}$

(29) A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are ω, v, α & a_c. Which of the following relations is incorrect ?

(a) $\omega \perp v$ (b) $\omega \perp a_c$

(c) $\omega \perp \alpha$ (d) $v \perp a_c$

(30) The equation of motion of a particle moving on a circular path (radius 200 m) is $s = 18t + 3t^2 - 2t^3$ where s is distance covered from a point in metre at the end of t second. The maximum speed of particle will be .....

(a) 15 m/s (b) 23 m/s

(c) 19.5 m/s (d) 25 m/s

(31) A body revolves n times in a circle of radius **p** cm in one minute. Its linear velocity is

(a) $\dfrac{60}{2n}$ cm/s (b) $\dfrac{2n}{60}$ cm/s

(c) $\dfrac{2\pi^2 n}{60}$ cm/s (d) none of these

(32) The moon is about 3.8×10^5 km from the centre of the earth. It takes about 27 days for completing the orbit around the earth. The speed of moon in km per day is ...

(a) 8.8×10^4 km/day

(b) 8.5×10^6 km/day

(c) 8.8×10^7 km/day

(d) 9×10^9 km/day

1.4 UNIFORM CIRCULAR MOTION (U.C.M) :

(33) The variable quantity when a body performs uniform circular motion in horizontal plane is ..

(a) speed

(b) linear momentum

(c) kinetic energy

(d) angular momentum

(34) A body moving in a circle at constant speed has an velocity which is constant in,

(a) magnitude only

(b) direction only

(c) both magnitude and direction

(d) none of these

(35) A particle is performing uniform circular motion, has constant

(a) velocity (b) kinetic energy

(c) momentum (d) acceleration

(36) If a particle convers half the circle of radius R with constant speed then,

(a) change in momentum is mvr

(b) change in K.E. is $\frac{1}{2}mv^2$

(c) change in K.E. is mv^2

(d) change in K.E. is zero

(37) The uniform circular motion is acceleration motion, because

(a) the motion accelerates due to the change in velocity

(b) the motion acceleration due to the change in angular velocity

(c) the motion accelerates due to the force

(d) all of these

(38) An object moves along a curved path. The following quantities may remain constant during its motion

(a) speed

(b) velocity

(c) magnitude of acceleration

(d) Both 'a' and 'c'

(39) In uniform circular motion, the angle a circle, in the clockwise direction. If it starts moving in the anticlockwise direction, then

(a) the centripetal force will be doubled

(b) the direction of the centripetal force will be reversed

(c) there will be no change in the magnitude and direction of the centripetal force

(d) the centrifugal force will act towards the centre

1.5. ACCELERATION IN UNIFORM CIRCULAR MOTION (RADIAL ACCELERATION) :

(40) A motor car is travelling 20 m/s on a circular road of radius 400 m. If it increasesa its speed at the rate of 1 m/s^2, then its acceleration will be

(a) $2\sqrt{2}\ m/s^2$ (b) $\sqrt{3}\ m/s^2$

(c) $\sqrt{2}\ m/s^2$ (d) $3\sqrt{3}\ m/s^2$

(41) If a particle moves in a circle of radius 25 cm at 2 rps, then the acceleration of the particle in m/s^2 will be

(a) $12\pi^2$ (b) $8\pi^2$ (c) $4\pi^2$ (d) $2\pi^2$

(42) The vector relation among radial acceleration, angular velocity and linear velocity is

(a) $\vec{a}_r = \vec{\omega} \cdot \vec{v}$ (b) $\vec{a}_r = \vec{\omega} \times \vec{v}$

(c) $\vec{a}_r = \vec{v} \times \vec{\omega}$ (d) $\vec{a}_r = \vec{v} \cdot \vec{\omega}$

(43) The velocity of a body of mass 2 kg moving in circle of radius 3 m at any time is 3 m/sec. If its speed is increasing at the rate of 4 m/sec^2 then the net acceleration on the body is

(a) 4 m/sec^2 b) 7 m/sec^2

(c) 3 m/sec^2 d) 5 m/sec^2

(44) A body is revolving with a uniform speed v in a circle of radius r. The tangential acceleration is

(a) v/r (b) v^2/r (c) Zero (d) v/r^{2-}

(45) A particle is performing non UCM, the acceleration of the particle is $\vec{a_R} = \vec{a_r} + \vec{a_T}$, where $\vec{a_r}$ is radial component of acceleration and $\vec{a_T}$ is tangential component of acceleration. If $\vec{a_r} = 0$, the motion of the particle is

(a) uniform circular motion

(b) non uniform circular motion

(c) straight line motion along the tangent to cuve path

(d) spiral motion about centre

(46) A stone tied to the end of a string 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44s, what is the magnitude and direction of acceleration of the stone?

(a) $\pi^2(m/s^2)$ and direction along the tangent to the circle.

(b) $\pi^2(m/s^2)$ and direction along the radius towards the centre.

(c) $\pi^2/4(m/s^2)$ and direction along the radius towards the centre.

(d) π^2 (m/s^2) and direction along the radius away from the centre.

(47) If a_r and a_t represents radial and tangential accelerations, the motion of a particle will be uniform circular if

(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ but $a_t \neq 0$

(c) $a_r \neq 0$ but $a_t = 0$ (d) $a_r \neq 0$ and $a_t \neq 0$

(48) The speed of revolution of a particle going around a circle is halved and its angular speed is doubled. The centripetal acceleration ...

(a) remains unchanged

(b) halved

(c) doubled

(d) becomes four times

(49) Two bodies of mass 10 kg and 5 kg moving in concertric orbits of radii R and r such that their periods are the same. The ratio of their centripetal acceleration is ....

(a) R/r (b) r/R

(c) R^2/r^2 (d) r^a/r^2

(50) An aeroplane is moving on a circular path with a uniform speed 300 km/h. If the period of the aeroplane is 12 hours, then the average acceleration after half cycle will be

(a) zero (b) 100 km/h²

(c) 50 km/h² (d) 25 km/h²

(51) A car is moving with speed 30 ms^{-1} on a circular path of radius 500 m. Its speed is increasing at a rate of 2 ms^{-2}, what is the acceleration of the car ?

(a) 2 ms^{-2} (b) 2.7ms^{-2}

(c) 1.82 ms^{-2} (d) 9.82 ms^{-2}

1.6 CENTRIPETAL AND CENTRIFUGAL FORCES :

(52) A small coin is kept at the rim of a horizontal circular disc, which is set into rotation about vertical axis passing through its centre. If radius of the disc is 5 cm and $\mu_s = 0.25$, then the angular speed at which the coin will just slip is

(a) 5 rad/s (b) 7 rad/s

(c) 10 rad/s (d) 4.9 rad/s

(53) A small coin is placed on a turntable at a certain distance from the axis of rotation. The coin begins to slide just as the turn table reaches a speed of 60 r.p.m. If another similar coin is stuck on top of first coin, then the sliding would commence at a speed of

(a) 45 r.p.m. (b) 90 r.p.m.

(c) 30 r.p.m. (d) 60 r.p.m.

(54) A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes in a plane, then ......

(a) its velocity is constant.

(b) its acceleration is constant.

(c) its kinetic energy is constant.

(d) it moves in a straifght line

(55) A string can withstand a tension of 25 N. The great speed at which a body of mass 1 kg can be whirled in a horizontal circle using 1 m length of the string is

(a) 10 m/sec (b) 5 m/sec

(c) 0.5 m/sec (d) none.

(56) A car is moving on a circular track of diameter 72 m with a speed of 6m/s. It is accelerated at the rate of $\sqrt{3}\,m/s^2$. If the mass of the car is 1000 kg, the net force acting on the car is

(a) 1000 N (b) 2000 N

(c) $1000\sqrt{3}$ N (d) $\frac{1000}{\sqrt{3}}$ N

(57) The centripetal force required to hold 1 kg object in circular path by means of a stirng 1 m long if the object is moving at constant speed of 2 m/s will be

(a) 2 N (b) 8 N (c) 4 N (d) 12 N

(58) The change in the centripetal force of a body moving in a circular path, if speed is made half and radius is made 4 times the original value, will

(a) increase by $\frac{16}{15}$ (b) decrease by $\frac{15}{16}$

(c) decrease by $\frac{8}{15}$ (d) increase by $\frac{8}{15}$

(59) A particle is revolving with a constant angular velocity along a circular path. If its direction of motion is reversed, keeping the angular velocity constant, then the centripetal force will :

(a) away from centre

(b) not change its magnitude

(c) towards centre

(d) both (b) and (c)

(60) A mass ' m' on a frictionless table is attached to a hanging mass 'M' by a chord through a hole in the table the condition with which 'm' must spin, for 'M' to stay at rest is :

(a) $\frac{v^2}{R} = \frac{m}{Mg}$ (b) $\frac{v^2}{R} = \frac{Mg}{m}$

(c) $\frac{v^2}{R} = \frac{m}{M}$ (d) none

(61) The centripetal foce in magnitude and direction is given by

(a) $\frac{mv^2}{r}\vec{r}$ (b) $-mr\omega^2 \vec{r}_0$

(c) $-\frac{mv^2}{r}\vec{r}_0$ (d) both 'b' and 'c'

(62) For keeping a body in uniform circular motion, the force required is

(a) Centrifugal (b) Radial

(c) Tangential (d) Centripetal

(63) A proton of mass 1.6×10^{-27} kg, revolves in a circular path of radius 0.1 m. If it is acted upon by a centripetal force of 4×10^{-13} N, then the angular velocity of the proton is

(a) 3×10^7 rad/s (b) 4×10^7 rad/s

(c) 5×10^7 rad/s (d) 8×10^7 rad/s

(64) Centrifugal force is

(a) a real force acting along the radius

(b) a force whose magnitude is less than that of the centripetal force

(c) a pseudo force acting along the radius and away from the centre

(d) a force which keeps the body moving along a circular path with uniform speed

(65) Due to centrifugal force acting on earth

(a) Results in bulging at the poles and flattening at equator

(b) Results in bulging at equator and flattening at the poles

(c) Bulging at both poles and equator

(d) Flattening at both poles and equator

(66) Which one of the following forces is a pseudo force ?

(a) Force acting on a faliling body

(b) Force acting on a charged particle placed in an electrical field

(c) Force experienced by a person standing on a merry go-round

(d) Force which keeps the electrons moving in circular orbits

(67) A particle of mass 'm' moves with a constant speed along a circular path of radius r under the action of a force F. Its speed is given by

(a) $\sqrt{\frac{Fr}{m}}$ (b) $\sqrt{\frac{F}{mr}}$ (c) $\sqrt{\frac{F}{r}}$ (d) $\sqrt{Fmr}$

(68) For a body moving along a circular path, the condition for no skidding of is

(a) $\frac{mv^2}{r} \geq \mu mg$ (b) $\frac{mv^2}{r} \leq \mu mg$

(c) $\frac{mv^2}{r} = \mu mg$ (d) $v = r\mu g$

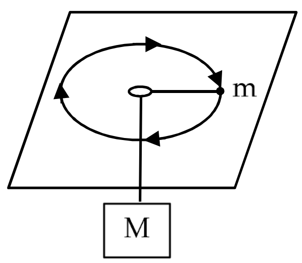
(69) On a dry road, the maximum permissible speed of a car in a circular path is 10 ms^{-1}. If the road becomes wet, then the maximum speed is $5\sqrt{2}$ ms^{-1}. If the coefficient of friction for dry road is m, then that for the wet road is

(a) $\frac{\mu}{2}$ (b) $\frac{\mu}{3}$ (c) $\frac{2\mu}{3}$ (d) $\frac{3\mu}{4}$

(70) A person stands in contact against a wall of cylindrical drum of radius 'r' rotating with an angular velocity ω. If μ is coefficient of static friction between the wal and the person, then the minimum rotational speed which enables the person to remain stuck to the wall will be

(a) $\sqrt{\frac{g}{\mu r}}$ (b) $\sqrt{\frac{\mu r}{g}}$ (c) $\sqrt{\frac{2g}{\mu r}}$ (d) $\sqrt{\frac{gr}{\mu}}$

(71) A mass m on a friction less able is attached to a hanging mass M by a cord through a hole in the table. Then the angular speed with which m must spin for M to stay at rest will be,



(a) $\sqrt{\frac{Mg}{mr}}$ (b) $\sqrt{\frac{mg}{Mr}}$ (c) $\sqrt{\frac{mr}{Mg}}$ (d) $\sqrt{\frac{g}{r}}$

(72) A mass is supported on frictionless smooth horizontal surface. It is attached to a string rotated about a fixed centre at an angular velocity ω_0. If the length of the string and the angular velocity are doubled, the tension in the string which was originally T_0, is now

(a) T_0 (b) $T_0/2$ (c) $4T_0$ (d) $8T_0$

(73) Two identical particles A and B are situated respectively at the midpoint and at the end of a string. The particles always remain collinear and move in concentric circles. The ratio of the tensions T_1 and T_2 will be

(a) 1 : 1 (b) 1 : 3 (c) 2 : 3 (d) 3 : 2

1.7 BANKING OF ROADS :

(74) When high speeds are normally used on curved roads, the roads are frequently banked (i.e. the outer part of the road is built at a higher level than the inner part) so that

(a) there is no friction between the road and the tyres.

(b) the weight of the automobiles may be reduced.

(c) the necessary centripetal force to make the automobiles more in the circular path may be obtained from the horizontal component of the normal reaction.

(d) none of these

(75) A simple pendulum of length 1 m, the bob performs circular motion in horizontal plane if its string making an angle 60^0 with the vertical, then the period of rotation of the bob will be ($g = 10\ m/s^2$)

(a) 2 s (b) 1.4 s (c) 1.98 s (d) 4 s

(76) A motor cyclist moving with a velocity of 72 km per hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 metres. The acceleration due to gravity is 10 m/sec^2. In order to avoid skidding he must not bend with respect to the vertical plane by an angle greater than

(a) $\theta = \tan^{-1}(6)$ (b) $\theta = \tan^{-1}(25.92)$

(c) $\theta = \tan^{-1}(2)$ (d) $\theta = \tan^{-1}(4)$

(77) A car takes a turn on a slippery road at a safe speed of 9.8 m/s. If the coefficient of friction is 0.2, the minimum radius of the arc in which the car takes a turn is

(a) 20 m (b) 49 m (c) 24.5 m (d) 80 m

(78) Length of a simple pendulum is 2 m and mass of its bob is 0.2 kg. If the tension in the string exceeds 4 N, it will break. If $g = 10\ m/s^2$ and the bob is whirled in a horizontal plane, the maximum angle through which the sting can make with vertical during rotation is

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

(79) A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb line is suspended from the roof of the car by a light rod of length 1 m. What is the angle made by the rod with the vertical ? [g = 10 m/s²]

(a) 30^0 (b) 45^0 (c) 60^0 (d) 0^0

(80) A car is moving in a horizontal circular track of radius 10 m, with a constant speed of 36 km/hour. A simple pendulum is suspended from the roof of the car. If the length of the simple pendulum is 1 m, what is the angle made by the string with the track ?

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(81) On a banked road, the component R cos θ of the resultant reaction balances

(a) the centrifugal force

(b) the weight of the car

(c) the frictional force

(d) the centripetal force

(82) The radius of curvature of a metre guage railway line at a place, where the train is moving with a speed of 10 m/s is 50 m. If there is no side thrust on the rails, than the elevation of the outer rail above the inner rail is

(a) 0.1 m (b) 0.2 m (c) 0.3 m (d) 0.4 m

(83) What will be the maximum speed of a car on a road turn of radius 30 m, if the coefficient of friction between the tyres and the road is 0.4 ? (Take g = 9.8 m/s²)

(a) 10.84 m/s (b) 9.84 m/s

(c) 8.84 m/s (d) 6.84 m/s

(84) In a conical pendulum, when the bob moves in a horizontal circle of radius r, with uniform speed v, the string of length L describes a cone of semivertical angle θ. The tension in the string is given by

(a) $T = \dfrac{mgL}{\left(L^2 - r^2\right)}$ (b) $\dfrac{\left(L^2 - r^2\right)^{1/2}}{mgL}$

(c) $T = \dfrac{mgL}{\sqrt{L^2 - r^2}}$ (d) $T = \dfrac{mgL}{\left(L^2 - r^2\right)^2}$

1.8 VERTICAL CIRCULAR MOTION DUE TO EARTH'S GRAVITATION :

(85) Vertical circular motion is

(a) non-uniform circular motion

(b) uniform circular motion

(c) both 'a' and 'b'

(d) none of the above

(86) A particle moving in a vertical circle its

(a) kinetic energy is constant

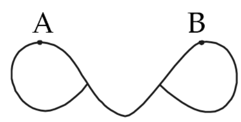
(b) potential energy is constant

(c) neither K.E. nor P.E. constant

(d) both kinetic energy and potential eneryg constant

1.9 EQUATION FOR VELOCITY AND ENERGY AT DIFFERENT POSITIONS IN VERTICAL CIRCULAR MOTION:

(87) A car moves at a constant speed on a road as shown in figure below. The normal force by the road on the car is N_A and N_B when it is at the points A and B respectively.



(a) $N_A = N_B$

(b) $N_A > N_B$

(c) $N_A < N_B$

(d) insufficient information to decide the relation of N_A and N_B.

(88) A small pot completely filled with water is tied at the end of a 1.6 m long string. It is whirled in a vertical circle. What minimum speed should be given to the pot, so that the water from the pot does not spill when the pot is at the highest position ? (use g = 10 m/s²)

(a) 2 m/s (b) 4 m/s

(c) 8 m/s (d) 16 m/s

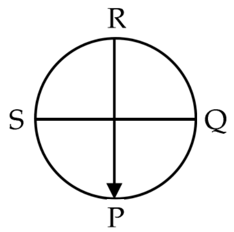
(89) A roller coaster is designed such that riders experience 'weightlessness' as they go round the top of a hill whose radius of curvature is 20 m. The speed of the car at the top of the hill is between

(a) 14 m/s and 15 m/s

(b) 15 m/s and 16 m/s

(c) 16 m/s and 17 m/s

(90) A stone is attached to one end of a string and rotated in a vertical circle. If the string breaks at the position of maximum tension, then it will break at



(a) Q (b) P (c) S (d) R

(91) A small body attached at the end of an inextensible string completes a vertical circle, then its

(a) angular velocity remains constant

(b) angular momentum remains constant

(c) total mechanical energy remains constant

(d) linear momentum remains constant

(92) Kinetic energy of a body moving in vertical circle is

(a) constant at all points on a circle

(b) different at different points on a circle

(c) zero at all the point on a circle

(d) negative at all the points

(93) In a roller coaster, car slows down and speeds up as it moves around vertical loop due to

(a) change of velocity at lowest and highest point

(b) uniform circular motion

(c) vertical circular motion

(d) both 'a' and 'c'

(94) A stone is tied to a string of length L and is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in velocity as it reaches a horizontal position where the string is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2\left(u^2 - gL\right)}$

(95) A moton cyclist rides around the well with a round vertical wall and does not fall down while riding because

(a) the force of gravity disappears

(b) the frictional force of the wall balances his weight

(c) he loses weight somehow

(d) the force exerted by the surrounding

(96) A small body of mass 'm' slides without friction from the top of a hemispherical bowl of radius 'r'. The vertical distance covered by it below the highest point just before breaking off from the surface is 'h'. The ratio of h to r is.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{7}$

(97) A particle performs vertical circular moiton along the circular path. If the ratio of kinetic energy to potential energy of a particle at any psotion is (If the particle makes an angle θ with vertical at the position)

(a) $\frac{3+2\cos\theta}{1-\cos\theta}$ (b) $\frac{1}{2}\frac{\left(3+2\cos\theta\right)}{\left(1-\cos\theta\right)}$

(c) $\frac{1-\cos\theta}{3+2\cos\theta}$ (d) $\frac{1+\cos\theta}{3-2\cos\theta}$

(98) A bucket full of water is rotated in vertical circle of radius 20 m. The minimum speed that the bucket should have so that water will not fall when it is at the highest point is (g = 9.8 m/s²)

(a) $\sqrt{98}$ m / s (b) $\sqrt{9.8}$ m / s

(c) 104 m/s (d) 1.4 m/s

(99) A 1 kg stone at the end of 1 m long string is whirled in a vertical circle at constant speed of 4 m/s. The tension in the string is 6 N, when the stone is at

(a) top of the circle (b) bottom of the circle

(c) half way down (d) none of the above

(100) A bucket filled with water is revolved in a vertical circle of radius 4 m. If g = 10m/s², the time period of revolution should be less than

(a) 10 s (b) 8 s (c) 5 s (d) 4 s

(101) A mass m is hanging by a string of length l. The velocity v_0 which must be imparted to it to just reach the top is

(a) $\sqrt{2gl}$ (b) $\sqrt{4gl}$ (c) $\sqrt{5gl}$ (d) $\sqrt{6gl}$

(102) A body of mass m hangs at one end of a string of length l, the other end of which is fixed. If is given a horizontal velocity so that the string would just reach an angle of 60^{o} with the vertical. The tension in the string at mean position is

(a) 2 mg (b) mg (c) 3 mg (d) $\sqrt{3}$ mg

(103) A body of mass m is revolving along a vertial circle of radius r such that the sum of its kinetic energy and potential energy is constant. If the speed of the body at the highest point is $\sqrt{2rg}$ then the speed of the body at the lowest point is

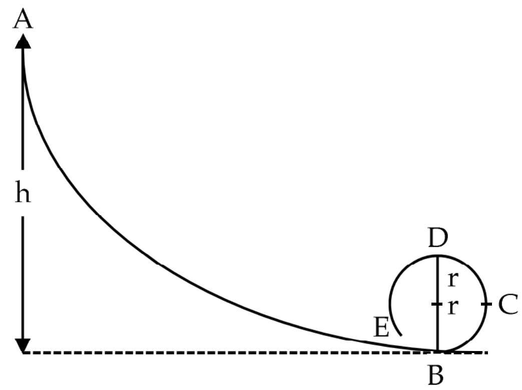
(a) $\sqrt{4gr}$ (b) $\sqrt{6gr}$ (c) $\sqrt{2gr}$ (d) $\sqrt{gr}$

(104) A bucket full of water is rotated in vertical circle of radius 20 m. The minimum speed that the bucket should have so that water will not fall when it is at the highest point is (g = 9.8 m/s²)

(a) $\sqrt{98}$ m / s (b) $\sqrt{9.8}$ m / s

(c) 104 m/s (d) 1.4 m/s

(105) A frictionless track ABCDE ends in a circular loop of radius 'r'. A body slides down the track from the point 'a' which is at a height h = 10 cm. The maximum value of 'r' for the body to successfully complete the loop is



(a) 2 cm (b) 1 cm (c) 4 cm (d) 6 cm

(106) A 4 kg ball swings in a vertical circle at the end of chord 1 m long. The maximum speed at which it can swing if chord can sustain maximum tension of 140 N is (g = 10 m/s²)

(a) 2 m/s² (b) 3 m/s

(c) 4 m/s (d) 5 m/s

(107) A body of mass 'm' is rotated by means of a string along a vertical circle of radius 'r' with constant speed. The difference in tensions when the body is at the bottom and at the top of the vertical circle is

(a) 6 mg (b) 4 mg (c) 2 mg (d) zero

(108) A simple pendulum of mass and length l stants in equilibrium in vertical position. The maximum horizontal velocity that should be given to the bob at the bottom so that it completes one revolution is

(a) $\sqrt{lg}$ (b) $\sqrt{2lg}$ (c) $\sqrt{3lg}$ (d) $\sqrt{5lg}$

(109) At any instant the tension T along the length of string during the course of oscillation of simple pendulum is given by

(a) $T = mg\cos\theta + (mv^2/r)$

(b) $T = mg\sin\theta + (mv^2/r)$

(c) $T = mg\cos\theta - (mv^2/r)$

(d) none of these

(110) A stone of mass m is tied to a string of length L and moved in a vertical circle at the rate of n revolutions/minute. The tension in the string when it is at its lowest point is

(a) $m[g + 4\pi^2 r]$ (b) $m\left[g + \frac{\pi^2 n^2 r^2}{60}\right]$

(c) $m\left[g + \frac{\pi^2 n^2 r^2}{900}\right]$ (d) $m[g + n^2 r^2]$

(111) A mass m is kept hanging by a rod of length L. What tangential velocity must be given to it so that it can just reach the top of the vertical circle ?

(a) $5\sqrt{gL}$ (b) $4\sqrt{gL}$ (c) $3\sqrt{gL}$ (d) $2\sqrt{gL}$

(112) A body of mass 1 kg is suspended from a string 1 m long. It is rotated in a vertical circle. What is the tension in the string, when it is horizontal and the speed of the body is 2 m/s ?

(a) 4 N (b) 3 N (c) 2 N (d) 1 N

(113) A weightless thread can bear a maximum tension of 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a vertical circular path of radius 2 m in a vertical plane. If $g = 10 m/s^2$, then the maximum angular velocity of the stone will be

(a) 3 rad/s (b) 4 rad/s

(c) 5 rad/s (d) 6 rad/s

(114) A bucket full of water is revolved in a vertical in circle of radius 1 m. When is the minimum frequency of revolution, required to prevent the water from falling down ? [$g = 10 m/s^2$]

(a) $\frac{\sqrt{10}}{2\pi}$ Hz (b) $\frac{2\pi}{\sqrt{5}}$ Hz

(c) $\frac{\sqrt{5}}{\pi}$ Hz (d) $\frac{2\pi}{\sqrt{10}}$ Hz

(115) A body of mass 1 kg is rotating in a vertical circle of radius 1 m. What will be the difference in its kinetic energy at the top and bottom of the circle ?

($g = 10 m/s^2$)

(a) 10 J (b) 30 J (c) 20 J (d) 50 J

(116) A pendulum consisting of a small sphere of mass m, suspended by a inextensible and in a vertical plane. If the breaking strength of the string is 2 mg, then the maximum angular amplitude of the displacement from the vertical can be

(a) 90° (b) 60° (c) 30° (d) 0°

1.10 KINEMATICAL EQUATIONS FOR CIRCULAR MOTION IN ANALOGY WITH LINER MOTION :

(117) What is the angular acceleration of a particle in circular motion, which slows down from 600 r.p.m. to rest in 10s?

(a) -2π rad/s² (b) $-\pi$ rad/s²

(c) -3π rad/s² (d) $-\frac{\pi}{2}$ rad/s²

(118) The speed of a mototr increases from 1200 rpm to 1800 rpm in 20 s. How many revolutions does it make during these second ?

(a) 400 (b) 600

(c) 500 (d) 700

(119) When a ceiling fan is switched off, its angular velocity falls to half while it makes 36 rotations. How many more rotaions will it make before coming to rest ?

(a) 24 (b) 36 (c) 18 (d) 12

(120) A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first two seconds it rotates through an angle θ_1. In the next two second it rotates through an angle θ_2. What is the ratio θ_2/θ_1 ?

(a) 1 (b) 2 (c) 3 (d) 4

(121) A wheel starts from rest and acquires an angular velocity of 60 rad/s in half a minute. Then its angular acceleration is

(a) 4 rad/s^2 (b) 2 rad/s^2

(c) 1 rad/s^2 (d) 0.5 rad/s^2

(122) A car accelerates uniformly from rest to a speed of 10 m/s in a time of 5 s. The number of revolutions made by one of its wheels during the motion if the radius of the wheel is $1/\pi$ m.

(a) 50 (b) 25 (c) 12.5 (d) 6.25

(123) The car of a wheel rotating with certain angular velocity is stopped in 7 seconds and before it stops, it makes 35 revolutions. Then initially it was rotating with the frequency.

(a) 10 Hz (b) 20 Hz (c) 15 Hz (d) 30 Hz

(124) The angular velocity of a particle increases from O to ω as it complexes x rotations. Then number of rotations completed by it when its angular velocity become 2ω.

(a) x (b) 2x (c) 3x (d) 4x

(125) An automobile engine starting from rest is given an angular acceleration of 20 rad/s^2 for 10 s. Find the angle turned during this period

(a) 10 rad (b) 100 rad

(c) 1000 rad (d) 0.1 rad

(126) A flywheel is revolving at 150 revolutions per minute. If it deccelerates at a constant rate of 2π rad/s^2, then time required to stop it is

(a) 10 s (b) 5 s (c) 2.5 s (d) 1.25 s

CLASS WORK - ANSWER KEY

1 **b**	2 **d**	3 **b**	4 **b**	5 **b**	6 **d**	7 **c**	8 **d**	9 **c**	10 **b**
11 **b**	12 **b**	13 **a**	14 **a**	15 **b**	16 **c**	17 **b**	18 **a**	19 **d**	20 **c**
21 **b**	22 **c**	23 **b**	24 **c**	25 **d**	26 **c**	27 **a**	28 **b**	29 **d**	30 **a**
31 **b**	32 **a**	33 **b**	34 **b**	35 **d**	36 **d**	37 **c**	38 **d**	39 **c**	40 **b**
41 **d**	42 **c**	43 **c**	44 **c**	45 **a**	46 **c**	47 **b**	48 **b**	49 **b**	50 **c**
51 **d**	52 **b**	53 **b**	54 **a**	55 **a**	56 **d**	57 **d**	58 **a**	59 **b**	60 **d**
61 **a**	62 **a**	63 **c**	64 **b**	65 **b**	66 **b**	67 **b**	68 **c**	69 **b**	70 **b**
71 **a**	72 **d**								

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HOME WORK - ANSWER KEY

1 **b**	2 **d**	3 **a**	4 **a**	5 **b**	6 **a**	7 **a**	8 **c**	9 **c**	10 **b**
11 **b**	12 **d**	13 **d**	14 **d**	15 **b**	16 **b**	17 **d**	18 **c**	19 **d**	20 **c**
21 **c**	22 **d**	23 **c**	24 **a**	25 **c**	26 **b**	27 **b**	28 **d**	29 **c**	30 **c**
31 **c**	32 **a**	33 **b**	34 **a**	35 **b**	36 **d**	37 **a**	38 **d**	39 **c**	40 **c**
41 **c**	42 **b**	43 **d**	44 **c**	45 **c**	46 **b**	47 **c**	48 **a**	49 **a**	50 **b**
51 **b**	52 **b**	53 **d**	54 **c**	55 **b**	56 **b**	57 **c**	58 **b**	59 **d**	60 **c**
61 **d**	62 **d**	63 **c**	64 **c**	65 **b**	66 **c**	67 **a**	68 **b**	69 **a**	70 **a**
71 **a**	72 **d**	73 **d**	74 **c**	75 **b**	76 **c**	77 **b**	78 **c**	79 **b**	80 **b**
81 **b**	82 **b**	83 **a**	84 **c**	85 **c**	86 **c**	87 **c**	88 **b**	89 **a**	90 **b**
91 **c**	92 **b**	93 **d**	94 **d**	95 **b**	96 **a**	97 **b**	98 **c**	99 **a**	100 **d**
101 **c**	102 **a**	103 **b**	104 **c**	105 **c**	106 **d**	107 **c**	108 **d**	109 **a**	110 **c**
111 **d**	112 **a**	113 **c**	114 **a**	115 **a**	116 **b**	117 **d**	118 **c**	119 **d**	120 **c**
121 **b**	122 **c**	123 **a**	124 **d**	125 **c**	126 **c**				

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CLASSWORK

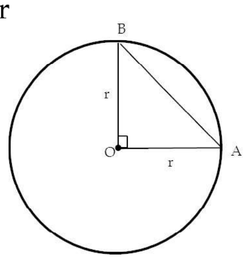
Hints & Explanation

1.1. ANGULAR DISPLACEMENT :

(1) (b) $0.5\ \pi r$, $\sqrt{2r}$

In quarter revolution.

distance = $\frac{\pi r}{2}$ = 0.5 πr



Displacement = l (AB)

$AB^2 = r^2 + r^2$

$AB^2 = 2r^2$

$AB = \sqrt{2}r$

$\therefore$ (b) is correct.

(2) (d) The instantaneous angular displacement and radius vectors are mutually parallel to each other.

The instantaneous angular displacement and radius vector are mutually perpendicular.

$\therefore$ (d) is correct

(3) (b) $\frac{2\pi}{3}$ radian

$$\theta = \omega t = \frac{2\pi}{60} \times 20 = \frac{2\pi}{3}$$

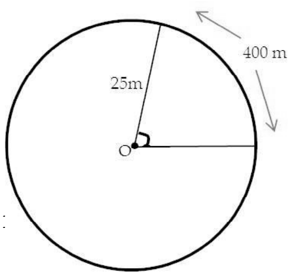
(4) (b) 16^c

r = 25m

s = 400 m

$$\theta = \frac{s}{r}\left(\frac{l(\text{arc})}{\text{radius}}\right)$$

$$= \frac{400}{25} = 16$$



θ = 16 radian

$\therefore$ (b) is correct

(5) (d) either 'b' or 'c'

Centripetal acceleration keeps on changing its direction.

1.2. ANGULAR VELOCITY AND ANGULAR ACCELERATION:

(6) (d) 12 : 1

$$\frac{\omega 1}{\omega 2} = \frac{2\pi}{10 \times 60} \times \frac{60}{2\pi} = \frac{1}{12}$$

(7) (c) both magnitude and direction

(8) (d) 60 radian

Comparing with $s = ut + \frac{1}{2}at^2$, we have

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 2 \times 5 + \frac{1}{2} \times 4 \times 25$$

$$= 10 + 50$$

$$= 60 \text{ radian}$$

(9) (c) $4\pi^2$

(10) (b) 3 cm

(11) (b) 17.5 rad/s²

$$\theta = 2t^3 - \frac{t^2}{4} + 4t$$

$$= 6t^2 - \frac{t^2}{2} + 4$$

$$\alpha = 12t - \frac{1}{2}$$

at 1.5 s,

$$\alpha = 12(1.5) - \frac{1}{2}$$

$$= \frac{35}{2} = 17.5 \text{rad}/s^2$$

(12) (b) 31.4 m/s

$V = r\omega$

$= r.2\ \pi n$

$$= \quad 0.5 \times 2\pi \times \frac{120}{60}$$

$$= \quad 4\pi \times 0.5$$

$$= \quad 2\pi \text{ m/s}$$

$$= \quad 6.28 \text{ m/s}$$

$\therefore$ (b) is correct

(13) (a) its motion is confirmed to a single plane

(14) (a) magnitude only

(15) (b) 2.9×10^{-6} m/s

We have to find linear speed at a distance of 0.5 cm from tip

So radius = 2.5 – 0.5 = 2cm

$$= \quad \frac{2}{100} \times \frac{2\pi}{60 \times 60 \times 12}$$

$$= \quad \frac{\pi}{180 \times 10^4}$$

$v \quad = \quad 2.9 \times 10^{-6}$ m/s

$\therefore$ (b) is correct.

1.3. RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY :

(16) (c) $\omega \quad = \quad \frac{v}{r}$

(17) (b) 466 m/s

$$v \quad = \quad r\omega \quad = \quad \frac{6400 \times 10^3 \times 2\pi}{86400} \quad = \quad 466 \text{ m/s}$$

$\therefore$ (b) is correct

(18) (a) 35 m/s

$v \quad = \quad r\omega \quad = \quad 0.5 \times 70 \quad = 35$ m/s

(19) (d) 0.1047 rad/sec ; 0.00314 m/sec

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.1047 \text{ rad/s}$$

$v = \omega r$

$= 0.1047 \times 3 \times 10^{-2} = 0.00314$m/sec

(20) (c) 45 m/s

$\omega = \alpha t = 3 \times 5 = 15$ rad/s

$\therefore v = r\omega = 3 \times 15 = 45$ m/s

(21) (b) π

1.4 UNIFORM CIRCULAR MOTION (U.C.M) :

(22) (c) Kinetic energy

$$\text{K.E} = \frac{1}{2} m\omega^2$$

which is scalar, so remains constant

(23) (b) both velocity acceleration change

(24) (c) 9.42 s

$$v = \frac{r\omega}{v}$$

$$\omega = \frac{v}{r} = \frac{7}{7} = 1$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi \text{ sec}$$

In half revolution, time taken $= \frac{2\pi}{2} = \pi$s

$\therefore$ In one and half revolution, time taken

$= 2\pi + \pi = 3\pi$s

$= 3 \times 3.14$

9.42 s

(25) d) all of the above

1.5. ACCELARATION IN UNIFORM CIRCULAR MOTION (RADIAL ACCERATION) :

(26) (c) $a = \sqrt{a_T^2 + a_T^2}$

(27) (a) the centripetal acceleration remains unchaned

(28) (b) $\sqrt{\left(\frac{v^4}{r^2} + a^2\right)}$

$$a = \sqrt{a_t^{\ 2} + b_r^{\ 2}}$$

$$\because a^r = \frac{v^2}{r}$$

and $a_t = a$

(29) (d) 800 : 1

$$a_T = r\alpha = 0.2\frac{(\omega_2 - \omega_1)}{t} = \frac{0.2(40-2)}{19} = 0$$

$$V_2 = r\omega_2 = 0.2 \times 40 = 8$$

$$a_r = \frac{V^2}{r} = \frac{64}{0.2} = 320$$

$$\frac{a_r}{a_T} = \frac{320}{0.4} = \frac{800}{1}$$

(30) (a) $-(8\ m/s^2)\,\hat{j}$

$$a = \frac{V^2}{R} - \frac{16}{2} = 8m/s^2$$

The acceleration is directed towards the centre.

∴ when object is at y = 2m, acceleration is $(8m/s^2) \wedge j$

1.6. CENTRIPETAL AND CENTRIFUGAL FORCES :

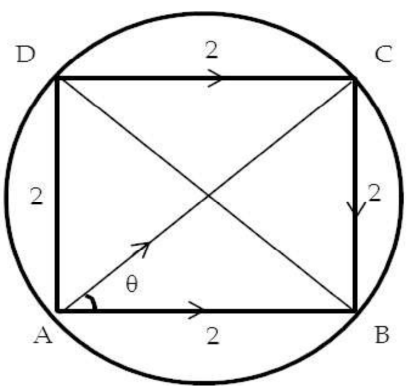
(31) (b) the centripetal force will not suffer any change in direction

(32) (a) 20 rad/s

$$T = mr\omega^2 \Rightarrow 10 = 0.25 \times \omega^2 \times 0.1 \Rightarrow \omega = 20rad/s$$

(33) (b) 246.8 N

Tension in a string is component of centripetal force along the of square.



$$\therefore T = C.\ P.\ force \times \cos\theta$$

$$= mr\omega^2 \cos 45$$

$$\text{where } r = \frac{AC}{2} = \frac{0.707}{2}$$

$$= 246.8N \quad = 1 \times \frac{0.707}{2} \times 4\pi^2.25 \times \frac{1}{\sqrt{2}}$$

(34) (b) the centrifugal force is more than the weight

(35) (d) 32 N

$$F = \frac{mv^2}{r}$$

$$= \frac{2 \times 4^2}{1}$$

$$= 2 \times 16$$

$$F = 32N$$

$\therefore$ (d)iscorrectoption of water

1.7. BANKING OF ROADS :

(36) (d) $\sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$

The forces acting on mass are, weight mg acting downward

centrifugal force = $\frac{mv^2}{r}$ acting radially downward.

Both forces are mutuallt perpendicular.

∴ Reading of spring balance.

$$= \sqrt{(mg^2 + \left(\frac{mv^2}{r}\right)^2}$$

∴ (d) is correct

(37) (c) to get the necessary centripetal force

(38) (d) 8.5m./sec

(39) (c) 24.2 m

(40) (b) $\sqrt{\mu gr}$

$$F = \frac{mv^2}{r} = \mu mg \therefore v = \sqrt{\mu gr}$$

(41) (d) 1, 2, 4

(d) is correct.

Third point os not corresponding to banking.

(42) (c) 6.28 s

Using $\tan\theta = \dfrac{v^2}{rg}$

$= \dfrac{10^2}{10\times10}$

= Time period of pendulum.

$$T = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$

$$= 2\pi\sqrt{\frac{10}{10\times1}}$$

$= 2\pi$

$= 2 \times 3.14 = 6.28$ s

(43) (c) $\theta = \tan^{-1}\left(\dfrac{1}{4}\right)$

$$v = \frac{90\times5}{18} = 25\text{m/s}$$

$$\tan\theta = \frac{v^2}{rg} = \frac{25\times25}{250\times10} = \frac{1}{4} \therefore \theta\tan^{-1}\left(\frac{1}{4}\right)$$

(44) (c) $mr\omega^2 = \mu mg$

(45) (a) $\dfrac{mgr}{\sqrt{L^2 - r^2}}$

$T\sin\theta = \dfrac{mv^2}{r}$, $T\cos\theta = mg$

$$\therefore \frac{\frac{mv^2}{r}}{mg} = \tan\theta$$

$$\therefore \frac{mv^2}{r} = mg\tan\theta = mg\frac{r}{\sqrt{L^2 - r^2}}$$

(46) (c) $\theta = \tan^{-1}(2)$

(47) (b) 2.8 rad/s

Centripetal force must be equal to frictional force for the coin to slip off,

$\therefore\ mr\omega^2 = \mu_s mg$

$$\therefore\ \omega = \sqrt{\frac{\mu_s g}{r}}$$

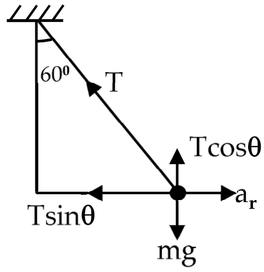
$$= \sqrt{\frac{0.2\times9.8}{0.25} = 2.8\text{rad/s}}$$

1.8. VERTICAL CIRCULAR MOTION DUE TO EARTH'S GRAVITATION :

(48) (b) 1.4 s

$T\sin\theta = m\omega^2 r = m\omega^2 / \sin q$ ....(i)

$T\cos\theta = mg$ ....(ii)



$\therefore$ From (i) and (ii), $\omega^2 = \dfrac{g}{l\cos\theta}$

$$\therefore\ \omega = \sqrt{\frac{g}{l\cos\theta}}$$

$\therefore$ Time period , $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

$= 2 \times 3.14 \times \sqrt{\dfrac{1\times\cos60^0}{10}}$

$= 1.4$ s

(49) (b) 1 N

$r = l\sin\theta$

$r = 10\sin30^0 \Rightarrow r = 5$ m , $T = 3$ s

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{3}$

Centripetal force $= m\omega^2 r$

$= 5\times10^{-2}\times\dfrac{4\pi^2}{9}$ ´ 5

$= 25\times10 - 2\times4$

$= 100\times10\text{–}2 \approx 1\text{N}$

(50) (c) 197 N

Tension in the string, $T = mr\omega^2 = mr(2\pi n)^2$

$= mr\,4\pi^2\,n^2$

$= 0.1\times(R+l)\times4\pi^2\times\dfrac{600}{60}$

$= 0.1 \times [1.5 \times 10^{-2} + 48.5 \times 10^{-2}] \times 4\pi^2 \times 10$

$= 197.9$ N

(51) (d) 60^0

Tension in the string is given by :

$T = mg\,[3 - 2\cos\theta]$

$4 = 0.2 \times 10\,[3 - 2\cos\theta]$

or, $\frac{4}{0.2\times10} - 3 = -2\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^0$

(52) (b) the centrifugal force is more than the weight of water

(53) (b) at the ends of th e horizontal diameter

(54) (a) the lowest point

(55) (a) acceleration due to gravity affects the motion of the body.

Whenever a body performs vertical circular motion. Acceleraton due to gravity affects the motion in a body. Hence, at every point, speed, kinetic energy, potential energy, tension changes but total energy remain constant.

1.9. EQUATION FOR VELOCITY AND ENERGY AT DIFFERENT POSITIONS IN VERTICAL CIRCULAR MOTION:

(56) (d) $\sqrt{2gl}$

$KE = \frac{1}{2} Mv^2$

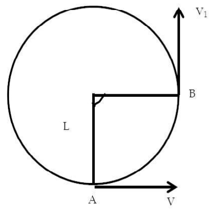
$\frac{1}{2} Mv^2 = mgl$

$V = \sqrt{2gl}$

(57) (d) $\sqrt{2\left(v^2 - gL\right)}$

$\frac{1}{2}mv^2 - \frac{1}{2}mv_1^{\,2} = mgL$

$\therefore v_1^{\,2} = v^2 - 2gL$



$\therefore v_1 = \sqrt{v^2 - 2gL}$

$\therefore |v - v_1| = \sqrt{v^2 + v^2} - 2gL = \sqrt{2(v^2 - gL)}$

(58) (a) $1.5\ ms^{-1}$

(59) (b) $\sqrt{2gr}$

$\frac{1}{2}mv^2 = mgr$

$\therefore v = \sqrt{2gr}$

(60) (d) $2\nu \sin 25^0$

(61) (a) $\frac{1}{2}mg\left(3\sqrt{3} - 2\right)N$

Let, $\theta_1 = 60^0$ and $\theta_2 30^0$

According to the law of conservation of energy,

$\frac{1}{2}mv^2 = mgl\,(\cos\theta_1 - \cos\theta_2)$

$\Rightarrow \frac{mv^2}{l} = 2\,mgl\,(\cos\theta_1 - \cos\theta_2)$

At 'B' tension $T = mg\cos\theta_2 + \frac{mv^2}{l}$

$= mg\cos\theta_2 + 2\,mg\,(\cos\theta_2 - \cos\theta_1)$

$= mg\,(3\cos\theta_2 - 2\cos\theta_1)$

$= mg\left(\frac{3\sqrt{3}}{2} - 1\right)$

$= \frac{1}{2}mg(3\sqrt{3} - 2)N.$

(62) (a) $T_2 - T_1 = 6\,mg$

$f_1 = \frac{mv^2}{r} - mg = 0$ and $T_2 = \frac{mv^2}{r} + mg = 6\,mg$

$\therefore \quad T_2 - T_1 = 6mg.$

(63) (c) $T_1 > T_2$

$$T = \frac{mv^2}{r} + mg\cos\theta$$

$$\text{At } \theta = 30^0,\ T_1 = \frac{mv^2}{r} + mg\cos s\ 50^0$$

$$\text{At } \theta = 60^0,\ T_2 = \frac{mv^2}{r} + mg\cos s\ 60^0$$

(64) (b) $2l$

The minimum velocity at B is $V_B = \sqrt{gl}$

Range will be

Range = $t \cdot V_B$

$$= \sqrt{\frac{2h}{1}}\sqrt{gl}$$

$$= \sqrt{\frac{2(20)}{1}}\sqrt{gl} = 21$$

(65) (b) $\sqrt{2gL}$

This is case of free end of ROD performing vertical circular motion.

When this free end strikes the ground, its velocity is same as velocity of midway position of particular.

$\therefore$ Velocity of rod = $\sqrt{2gL}$

(66) (b) 120 N, 60 N, 0 N

Tension at lowest point = 6mg
= 6 × 2 × 10
= 120 N

Tension at midway point = 3mg
= 3 × 2 × 10
= 60 N

Tension at highst point = 0 N

$\therefore$ 120 N, 60 N, N is proper order.

1.10. KINEMATICAL EQUATIONS FOR CIRCULAR MOTION IN ANALOGY WITH LINER MOTION :

(67) (b) 10

When fan is switched on

$$n_0 = 0, n = \frac{120}{60} = 2rps$$

$\therefore\ \omega = 2\pi n = 4\pi rad/s$

$\omega = 0$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\omega^2 - \omega^2{}_0}{2\theta}$$

$$\therefore \theta = \frac{\omega + \omega_0 t}{2} = 20\pi \text{ radian}$$

$$\therefore N = \frac{\theta}{2\pi} = 10 \text{ revolutions}$$

(68) (c) $\left(\sqrt{2} - 1\right)t$

(69) (c) 209 rad

$n_1 = 60$ rpm = 1 rps

$n_2 = 180$ rpm = 3 rps

$\omega_1 = 2\pi n_2 = 2\pi rad/s$

$\omega_2 = 2\pi n_2 = 2\pi \times 3 = 6\pi rad/s$

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{dt} = \frac{6\pi - 2\pi}{30} = \frac{4\pi}{30}$$

$$\alpha = \frac{4\pi}{30} \text{ rad/s}^2$$

Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

We get

$$\theta = 2\pi \times 20 = \frac{1}{2} \times \frac{4\pi}{30} \times (20)^2$$

= 125.6 = 83.73

θ = 209.3 rad

(70) (b) the acceleration is uniform

(71) (a) $100\ \pi$

(72) (d) 11 rad

$$\theta = \omega_1 t + \frac{1}{2}\alpha t_2 = 4 + 7 = 11 \text{ rad.}$$

HOME WORK

Hints & Explanation

1.1. ANGULAR DISPLACEMENT :

(1) (b) π

$$\theta = \omega t = \frac{2\pi}{T} \times \frac{T}{2}$$

$\therefore$ $\theta = \pi$

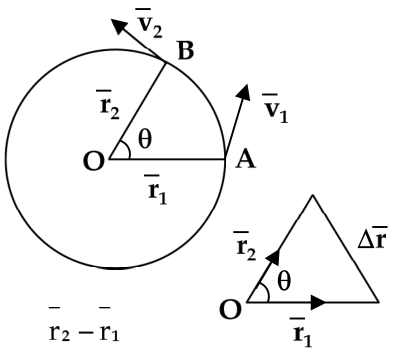
$\therefore$ (b) is correct

(2) (d) $\frac{\pi}{2}$ radian

(3) (a) it obeys the cumulative and associative laws of vector addition.

(4) (a) $2r \sin \frac{\theta}{2}$

The change in the position vector or displacement Δr of particle from position A to B is,



$$\Delta \bar{r} = \bar{r}_2 - \bar{r}_1$$

$$|\Delta \bar{r}| = |\bar{r}_2 - \bar{r}_1|$$

$$\Delta \bar{r} = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}$$

... (cosine rule)

$$= \sqrt{2r^2 - 2r^2 \cos\theta}$$

$$= \sqrt{2r^2 (1 - \cos\theta)}$$

$$= 2r \sin\frac{\theta}{2}$$

$\therefore$ (a) is correct

(5) (b) 1.14 r

Difference bteween linear distance and displacement $= \pi r - 2r$

$= r(\pi - 2)$

$= (3.14 - 2)\, r$

$= 1.14\, r$

$\therefore$ (b) is correct

(6) (a) it does not obey the law of vector adition.

(7) (a) 1.57 rad

In minute hand

$T = 60 \text{ min} = 60 \times 60$

$= 3600 \text{ sec}$

$t = 15 \text{ min} = 15 \times 60$

$= 900 \text{ s}$

$$\theta = \omega t = \frac{2\pi}{T} \times t$$

$$= \frac{2 \times 3.14 \times 900}{3600}$$

$$= \frac{2 \times 3.14}{4}$$

$$= 1.57 \text{ rad}$$

$\therefore$ (a) is correct

(8) (c) $10\,\pi$ rad

1.2 ANGULAR VELOCITY AND ANGULAR ACCELERATION:

(9) (c) 12 : 1

(10) (b) 1.047×10^3 rad/s^2

$n_1 = 200 \text{ rpm} = \frac{200}{60}$ rps

$n_2 = 400 \text{ rpm} = \frac{400}{60}$ rps

$dt = 20 \text{ ms} = 20 \times 10^{-3}$ s

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{dt}$$

$$= \frac{2\pi(n_2 - n_1)}{dt}$$

$$= 2 \times 3.14 \left(\frac{400-200}{60 \times 20 \times 10^{-3}}\right)$$

$$= 6.28 \times \frac{200 \times 10^3}{60 \times 20}$$

$$= \frac{6.28}{6} \times 10^3$$

$$= 1.047 \times 10^3 \text{ rad/s}^2$$

$\therefore$ (b) is correct

(11) (b) directed upwards for anticlockwise direction and downwards for clockwise direction

(12) (d) $\pi/2$ rad/s

(13) (d) 25 rad s^{-1}

(14) (d) 720 : 12 : 1

Angular velocity of second hand = 2p rpm

Angular velocity of minute hand = $\frac{\pi}{30}$ rpm

Angular velocity of hour hand = $\frac{\pi}{360}$ rpm

$\therefore$ Their ratio $= 2\pi : \frac{\pi}{30} : \frac{\pi}{360}$

$= 720 : 12 : 1$

$\therefore$ (d) is correct

(15) (b) 1 rad/s, 0.5 rad/s²

$$\frac{d\theta}{dt} = \frac{3t^2}{60} - \frac{1}{4} \quad \text{and}$$

$$\frac{d^2\theta}{dt^2} = \frac{6t}{60}$$

(16) (b) 2b + 6ct

$$\theta = at + bt^2 + ct^3$$

$$\therefore \frac{d\theta}{dt} = a + 2bt + 3ct^2$$

$$\frac{d^2\theta}{dt^2} = 2b + 3c \times 2t$$

$$= 2b + 6ct$$

(17) (d) π rad/s

$$\omega = \frac{2\pi}{T} \quad \text{and}$$

$$\omega = \frac{v}{r}$$

(18) (c) 10 rad/s²

(19) (d) 3π rad/s

$$\omega = 2\pi n = 2\pi \times \frac{3}{2}$$

$$= 3\pi$$

(20) (c) 24 radian/sec

$$\theta = 2t^3 + 0.5$$

$$\therefore \omega = \frac{d\theta}{dt} = 2 \times 3t^2$$

$$= 6t^2$$

(21) (c) 1 : 2

$$\omega_1 = \frac{v}{2r_1}, \quad \omega_2 = \frac{v}{r}$$

(22) (d) radial acceleration

(23) (c) radian /s²

(24) (a) zero

(25) (c) one

Since A, B and O always the lie along a straight line, their angular displacements are equal in equal intervals of time

$$\therefore \omega_1 = \omega_2 \quad \text{or} \quad \frac{\omega_1}{\omega_2} = 1$$

1.3 RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY :

(26) (b) 2.9×10^{-6} m/s

We have to find linear speed at a distance of 0.5 cm from tip.

So radius $= 2.5 - 0.5$

$= 2$ cm

$v = r\omega$

$= \frac{2}{100} \times \frac{2\pi}{60 \times 60 \times 12}$

$= \frac{\pi}{108 \times 10^4}$

$v = 2.9 \times 10^{-6}$ m/s

$\therefore$ (b) is correct

(27) (b) $\omega_1 > \omega_2$

r is decreased ω is increased.

(28) (d) $\vec{v} = \vec{\omega} \times \vec{r}$

(29) (c) $\omega \perp \alpha$

(30) (c) 19.5 m/s

$s = 18t + 3t^2 - 2t^3$

$v = \frac{ds}{dt}$

$v = 18 + 6t - 6t^2$

$\frac{dv}{dt} = 6 - 2t$

Velocity is maximum when $\frac{dv}{dt}$ is minimum

i.e. $\frac{dv}{dt} = 0$

$\therefore \quad 6 - 2t = 0$

$t = \frac{1}{2}$s

$v_{max} = 18 + 6 \times \frac{1}{2} - 6 \times \left(\frac{1}{2}\right)^2$

$= 19.5$ m/s

$\therefore$ (c) is correct

(31) (c) $\frac{2\pi^2 n}{60}$ cm/s

(32) (a) 8.8×10^4 km/day

Radius of orbit of moon = r = 3.8×10^5km

Time period of revolution of moon = T = 27 days.

Angular speed of moon.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{27}$ rad/day

Let v be speed of moon

$\therefore \quad v = r\omega$

$= 3.8 \times 10^5 \times \frac{2\pi}{27}$

$= 8.843 \times 10^4$ km/day

$\therefore$ (a) is correct

1.4 UNIFORM CIRCULAR MOTION (U.C.M) :

(33) (b) linear momentum

When a body is in uniform circular motion in a horizontal plane, its speed, angular velocity, period of rotation, kinetic energy and angular momentum are constant and its linear velocity, linear momentum, centripetal acceleration, centripetal force are variable.

$\therefore$ (b) is correct

(34) (a) magnitude only

(35) (b) kinetic energy

(36) (d) change in K.E. is zero

As momentum is vector quantity

$\therefore$ Change in momentum

$\Delta P = 2mv \sin\left(\frac{\theta}{2}\right)$

$= 2mv$

But K.E. always remains constant so change in K.E. is zero.

(37) (a) the motion accelerates due to the change in velocity

(38) (d) Both 'a' and 'c'

(39) (c) there will be no change in the magnitudeand direction of the centripetal force

1.5 ACCELARATION IN UNIFORM CIRCULAR MOTION (RADIAL ACCERATION) :

(40) (c) $\sqrt{2}\ m/s^2$

$$a_r = \frac{v^2}{r} = \frac{20\times 20}{400} = 1\ ;$$

$$a_T = 1$$

$$a_R = \sqrt{a_T^2 + a_r^2} = \sqrt{1+1}$$

$$= \sqrt{2}$$

(41) (c) $4\pi^2$

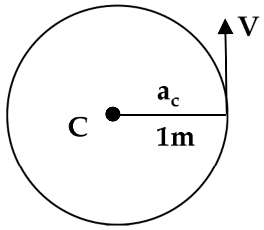
(42) (b) $\vec{a}_r = \vec{\omega} \times \vec{v}$

(43) (d) $5\ m/sec^2$

(44) (c) Zero

(45) (c) straight line motion along the tangent to cuve path

(46) (b) $\pi^2(m/s^2)$ and direction along the radius towards the centre.



The acceleration towards centre c

$$a_c = \omega^2 R$$

The tangential acceleration a_t is

$$a_t = \frac{dV}{dt} = 0$$

$\therefore$ Net acceleration $= \omega^2 R$

$$a_c = (2\pi n)^2 R = 4\pi^2 n^2 R$$

$$= 4\pi^2\left(\frac{22}{44}\right) \quad (1)$$

$$= \pi^2 \quad \text{towards centre}$$

$\therefore$ (b) is correct

(47) (c) $a_r \neq 0$ but $a_t = 0$

In uniform circular motion, tangential acceleration is zero but magnitude of radial acceleration is constant

(48) (a) remains unchanged

$$a = \frac{v^2}{r} = \frac{v\times v}{r} = \frac{v\times r\omega}{r} = v\omega$$

When v is halved and ω is doubled, a remains same as per above equation.

$\therefore$ (a) is correct

(49) (a) R/r

$$\frac{a_R}{a_r} = \frac{\omega_R^2 R}{\omega_r^2 r} = \frac{T_r^2}{T_R^2} = \frac{R}{r}$$

$$= \frac{R}{r} \quad (\because T_r = T_R)$$

(50) (b) $100\ km/h^2$

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{300-(-300)}{12/2}$$

$$= 100\ km/h^2$$

(51) (b) $2.7ms^{-2}$

1.6 CENTRIPETAL AND CENTRIFUGAL FORCES :

(52) (b) 7 rad/s

(53) (d) 60 r.p.m.

(54) (c) its kinetic energy is constant.

A force perpendicular to velocity changes only its direction and not magnitude (speed)

$\therefore$ (c) is correct

(55) (b) 5 m/sec

(56) (b) 2000 N

$$\frac{v^2}{r} = \frac{6\times 6}{36} = 1\ m/s^2$$

$\therefore \quad a_{net} = \sqrt{(a_R)^2 + (a_T)^2}$

$= \sqrt{1 + (\sqrt{3})^2}$

$= 2\ m/s^2$

$\therefore \quad F = 1000 \times 2$

$= 2000\ N$

(57) (c) 4 N

$F = \frac{mv^2}{r}$

(58) (b) decrease by $\frac{15}{16}$

(59) (d) both (b) and (c)

(60) (c) $\frac{v^2}{R} = \frac{m}{M}$

$Mg = \frac{mV^2}{r}$

$\frac{Mg}{m} = \frac{V^2}{r}$

(61) (d) both 'b' and 'c'

(62) (d) Centripetal

(63) (c) 5×10^7 rad/s

$F = \frac{mv^2}{r} = mr\omega^2$

$\therefore \quad \omega^2 = \frac{F}{mr} = \frac{4 \times 10^{-13}}{1.6 \times 10^{-27} \times 10^{-1}}$

$\therefore \quad \omega^2 = 25 \times 10^{14}$

$\therefore \quad \omega = 5 \times 10^7$ rad/s

(64) (c) a pseudo force acting along the radius and away from the centre

(65) (b) Results in bulging at equator and flattening at the poles

(66) (c) Force experienced by a person standing on a merry go-round

(67) (a) $\sqrt{\frac{Fr}{m}}$

$F = \frac{mv^2}{r}$

$\therefore \quad v = \sqrt{\frac{Fr}{m}}$

(68) (b) $\frac{mv^2}{r} \le \mu mg$

(69) (a) $\frac{\mu}{2}$

(70) (a) $\sqrt{\frac{g}{\mu r}}$

(71) (a) $\sqrt{\frac{Mg}{mr}}$

(72) (d) $8T_0$

$m_1 = m_2 = m$

$\frac{T_1}{T_2} = \frac{m_1}{m_2} \times \frac{r_1}{r_2} \times \frac{\omega_1^2}{\omega_2^2}$

$\frac{T_1}{T_2} = \frac{r}{2r} \times \frac{\omega_0^2}{4\omega_0^2}$

$\frac{T_1}{T_2} = \frac{1}{8}$

$T_2 = 8\ T_1$

(73) (d) 3 : 2

$N = \frac{n_1 + n_2}{2} \times t$

$= \frac{4 + 0}{2}$

1.7 BANKING OF ROADS :

(74) (c) the necessary centripetal force to make the automobiles more in the circular path may be obtained from the horizontal component of the normal reaction.

(75) (b) 1.4 s

(76) (c) $\theta = \tan^{-1}(2)$

(77) (b) 49 m

$$\frac{mv^2}{r} = \mu R$$

$$= \mu mg$$

$$\therefore \quad r = \frac{v^2}{\mu g} = \frac{9.8\times 9.8}{0.2\times 9.8}$$

$$= 49 \text{ m}$$

(78) (c) 60^o

$$T = mg(3 - 2\cos\theta)$$

$$= 6 - 4\cos\theta$$

$$\therefore \quad \cos\theta = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \quad \theta = 60^o$$

(79) (b) 45^0

$$\tan\theta = \frac{v^2}{rg} = 1$$

$$\therefore \quad \theta = 45^o$$

(80) (b) 45^0

$$\tan\theta = \frac{V^2}{rg} = \frac{10\times 10}{80\times 10}$$

$$= \frac{1}{8}$$

(81) (b) the weight of the car

(82) (b) 0.2 m

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \text{ and } h = l\sin\theta$$

$$= 0.2 \text{ m}$$

(83) (a) 10.84 m/s

(84) (c) $T = \dfrac{mgL}{\sqrt{L^2 - r^2}}$

For a conical pendulum, $T\cos\theta = mg$

$$\therefore \quad T = \frac{mg}{\cos\theta} = \frac{mg}{\frac{\left(L^2 - r^2\right)^{1/2}}{L}}$$

$$= \frac{mgL}{\sqrt{L^2 - r^2}}$$

1.8 VERTICAL CIRCULAR MOTION DUE TO EARTH'S GRAVITATION :

(85) (c) both 'a' and 'b'

(86) (c) neither K.E. nor P.E. constant

1.9 EQUATION FOR VELOCITY AND ENERGY AT DIFFERENT POSITIONS IN VERTICAL CIRCULAR MOTION:

(87) (c) $N_A < N_B$

(88) (b) 4 m/s

$$V_{min} = \sqrt{gr} = \sqrt{10\times 1.6}$$

$$= 4 \text{ m/s}$$

(89) (a) 14 m/s and 15 m/s

(90) (b) P

(91) (c) total mechanical energy remains constant

(92) (b) different at different points on a circle

(93) (d) both 'a' and 'c'

(94) (d) $\sqrt{2\left(u^2 - gL\right)}$

(95) (b) the frictional force of the wall balances his weight

(96) (a) $\dfrac{1}{3}$

(97) (b) $\dfrac{1}{2}\dfrac{\left(3 + 2\cos\theta\right)}{\left(1 - \cos\theta\right)}$

(98) (c) 104 m/s

At highest point in circular motion

$$v = \sqrt{rg} = \sqrt{20\times 9.8}$$

$= \sqrt{196}$

$= 14$ m/s

(99) (a) top of the circle

$mg = 1 \times 10 = 10$ N,

$\frac{mv^2}{r} = \frac{1\times(4)^2}{1} = 16$

Tension at the top of circle $= \frac{mv^2}{r} - mg$

$= 6$ N

Tension at the bottom of circle $= \frac{mv^2}{r} + mg$

$= 26$ N

(100) (d) 4 s

Since water does not fall, the velocity of revolution should be sufficient to provide centripetal acceleration at top of vertical circle.

$v = \sqrt{196} = \sqrt{10\times4}$

$= 2\sqrt{10}$

$\because T = \frac{2\pi r}{v} = \frac{2\times\pi\times4}{2\sqrt{10}}$

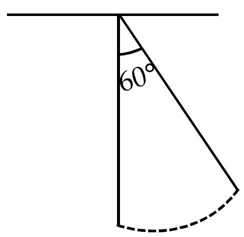
$= 4$s (approximately)

$\therefore$ (d) is correct

(101) (c) $\sqrt{5gl}$

(102) (a) 2 mg

When body is released from position P velocity is :



$v = \sqrt{2gl(1-\cos\theta)}$

Tension at lowest point

$= mg + \frac{mv^2}{l}$

$= mg + \frac{m}{l} \cdot 2gl(1 - \cos 60)$

$= 2$ mgl

$\therefore$ (a) is correct

(103) (b) $\sqrt{6gr}$

$v^2_b - v^2_h = 4gr$

$v^2_b - v^2_h = 4gr$

$\Rightarrow v_b = \sqrt{6gr}$

(104) (c) 104 m/s

At highest point in circular motion

$v = \sqrt{rg} = \sqrt{20\times9.8}$

$= \sqrt{196}$

$= 14$ m/s

(105) (c) 4 cm

$h = \frac{5r}{2} \Rightarrow r = \frac{2h}{5}$

$= \frac{2(10)}{5}$

$= 4$ cm

(106) (d) 5 m/s

Maximum tension at bottom is

$T = \frac{mv^2}{r} + mg$

$\therefore \frac{mv^2}{r} = T - mg$

$= 140 - 4 \times 10$

$\frac{4\times v^2}{1} = 100$

$v^2 = 25$

$v = 5\ m/s$

$\therefore$ (d) is correct.

(107) (c) 2 mg

$$T_{bottom} = \frac{mv^2}{r} + mg$$

$$T_{top} = \frac{mv^2}{r} - mg$$

$$T_{bottom} - T_{top} = 2mg$$

(108) (d) $\sqrt{5lg}$

(109) (a) $T = mg\cos\theta + (mv^2/r)$

(110) (c) $m\left[g + \frac{\pi^2 n^2 r^2}{900}\right]$

$$T = \frac{mv^2}{r} + mg$$

$$\frac{v^2}{r} = r\omega^2 = r4\pi^2\left(\frac{n}{60}\right)^2$$

$$= \frac{4\pi^2 n^2 r}{3600}$$

$$= \frac{\pi^2 n^2 r}{900}$$

$$\therefore \quad T = m\left[g + \frac{\pi^2 n^2 r}{900}\right]$$

(111) (d) $2\sqrt{gL}$

$$\frac{1}{2}mv^2 = mg\ (2L)$$

$$\therefore \quad v = 2\sqrt{gL}$$

(112) (a) 4 N

$$\text{Tension (T)} = \frac{mv^2}{r}$$

$$= \frac{1 \times 2^2}{1}$$

$$= 4\ N$$

(113) (c) 5 rad/s

The tension is maximum when the stone is at the lowest point.

Max. Tension

$$= \frac{mv^2}{r} + mg = mr\omega^2 + mg$$

$$\therefore \quad m[r\omega^2 + g] = T$$

$$\therefore \quad 0.5[2 \times \omega^2 + 10] = 30$$

$$\therefore \quad \omega^2 = 25$$

$$\therefore \quad \omega = 5\ rad/s$$

(114) (a) $\frac{\sqrt{10}}{2\pi}$ Hz

$$V_{min} = \sqrt{gr}$$

$$\therefore \quad V = r\omega = 2\pi fr$$

$$\therefore \quad f_{min} = \frac{V_{min}}{2\pi r} = \frac{\sqrt{gr}}{2\pi r}$$

$$= \frac{\sqrt{10}}{2\pi}$$

(115) (a) 10 J

$$V = \sqrt{\mu rg}$$

(116) (b) 60^o

1.10 KINEMATICAL EQUATIONS FOR CIRCULAR MOTION IN ANALOGY WITH LINER MOTION :

(117) (d) $-\frac{\pi}{2}\ rad/s^2$

$$a = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$$

$$= \frac{2\pi(0-10)}{10} = -2\pi\ rad/s^2$$

(118) (c) 500

$$\theta = 2\pi\frac{(n_1 + n_2)}{2}t$$

$$\therefore \quad \theta = 1000\ p$$

$$N = \frac{\theta}{2\pi}$$

(119) (d) 12

(120) (c) 3

(121) (b) 2 rad/s²

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{60}{30}$$

$$= 2\ \text{rad/sec}^2.$$

(122) (c) 12.5

$$n_2 = 0,$$

$$n_2 = \frac{\omega_2}{2\pi} = \frac{V}{2\pi \times r} = \frac{10 \times \pi}{1 \times 2\pi}$$

$$= 2\text{rps}.$$

$$N = \left(\frac{n_1 + n_2}{2}\right)t = \frac{5}{2} \times 5 = 12.5$$

(123) (a) 10 Hz

$$N = \frac{n_2 + n_2}{2} \times t$$

$$\therefore \quad n_1 = \frac{2N}{t} - n_2 = \frac{2 \times 35}{7} - 0$$

$$= 10\ \text{Hz}$$

(124) (d) $4x$

(125) (c) 1000 rad

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$\therefore \quad w^2 = 200\ \text{rad/sec}$$

$$n_2 = \frac{\omega_2}{2\pi} = \frac{200}{2\pi} = \frac{100}{\pi}$$

$$N = \frac{n_1 + n_2}{2} \times t$$

$$= \frac{50}{\pi} \times 10 = \frac{500}{\pi}$$

$$\therefore \quad \theta = \frac{500}{\pi} \times 2\pi = 1000\ \text{rad}$$

(126) (c) 2.5 s

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$\therefore \quad t = \frac{5\pi}{2\pi} = 2.5\ \text{sec}$$

Points to remember

- If the force acting on a particle is always perpendicular to the velocity of the particle, then the path of the particle is a circle. The centripetal force is always perpendicular to the velocity of the particle.
- If circular motion of the object is uniform, the object will possess only centripetal acceleration.
- If circular motion of the object is non-uniform, the object will possess both centripetal and transverse acceleration.
- When the particle moves along the circular path with constant speed, the angular velocity is also constant. But linear velocity, momentum as well as centripetal acceleration change in direction, although their magnitude remains unchanged.
- For circular motion of rigid bodies with uniform speed, the angular speed is same for all particles, but linear speed varies directly as the radius of the circular path described by the particle $(v \propto r)$.
- There can be no circular motion without centripetal force.
- Centripetal force cannot change the kinetic energy of the body.
- In uniform circular motion the magnitude of the centripetal acceleration remains constant whereas its direction changes continuously but always directed towards the centre.
- A pseudo force, that is equal and opposite to the centripetal force is called centrifugal force.
- The $\vec{\theta}, \vec{\omega}$ and $\vec{\alpha}$ are directed along the axis of the circular path. Their sense of direction is given by the right hand fist rule.
- $\vec{\theta}, \vec{\omega}$ and $\vec{\alpha}$ are called pseudo vectors or axial vectors.
- For circular motion we have :

  (i) $\vec{r} \perp \vec{v}$ (ii) $\vec{r}$ antiparallel to $\vec{a}_c$

  (iii) $\vec{a}_c \perp \vec{v}$ (iv) $\vec{a}_c \perp \vec{a}_t$

  (v) $\vec{\theta}, \vec{\omega}, \vec{\alpha}$ are perpendicular to $\vec{r}, \vec{a}_c, \vec{a}_t, \vec{v}$ (vi) $\vec{r}, \vec{a}_c, \vec{a}_t$ and $\vec{v}$ lie in the same plane

EVALUATION PAPER - CIRCULAR MOTION

Time : 30 Min. **Marks : 25**

(1) A stone tied to a string is whirled, then the string breaks at a certain speed because :
(a) Gravitational force is maximum
(b) The required centripetal force is greater than tension sustainable by the string
(c) Required centripetal force is equal to tension in the string
(d) Centripetal force is equal to weight

(2) The hour hand and the minute hand of a clock coincide at every relative periodic time in :
(a) 11/12 hour (b) 12/11 hours (c) 11/6 hours (d) 12/24 hour

(3) What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and half the angular speed ω ?
(a) the centripetal acceleration remains unchanged
(b) the centripetal acceleration is halved
(c) the centripetal acceleration is doubled
(d) the centripetal acceleration is quadrupled

(4) A coin placed on rotating turntable just slips, if it is placed at a distance of 4cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of
(a) 1 cm (b) 3 cm (c) 4 cm (d) 5 cm

(5) The angle θ in radians is
(a) $\left[M^0L^0T^1\right]$ (b) $\left[M^0L^0T^0\right]$ (c) $\left[M^0L^1T^{-1}\right]$ (d) $\left[M^0L^1T^1\right]$

(6) The frequencies of rotation of two particles in UCM of radii r_1 and r_2 on a circular disc of radius r in UCM are in the ratio
(a) 1 : 1 (b) $r_1 : r_2$ (c) $r_1^2 : r_2^2$ (d) $r_2 : r_1$

(7) A string can withstand a tension of 25 N. The maximum speed at which a body of mass 1 kg can be whirled in a horizontal circle using 1 m length of the string is
(a) 10 m/sec (b) 5 m/sec (c) 0.5 m/sec (d) none

(8) The average angular acceleration vector for a particle having a uniform circular motion is
(a) constant vector of magnitude $\dfrac{\sqrt{2}}{r}$
(b) null vector
(c) a vector of magnitude directed normal to plane of UCM
(d) equal to the instantaneous acceleration vector

(9) A body of mass 'm' is moving in a horizontal circle of radius 'r'. If the centripetal force is F, the kinetic energy of the body is

(a) $\frac{Fr}{2}$ (b) F r (c) $\frac{Fr^2}{2}$ (d) $\sqrt{Fr}$

(10) A simple pendulum is of length l. It is displaced so that its length becomes horizontal and then released then its velocity at bottom will be

(a) $\sqrt{gl}$ (b) $\sqrt{2gl}$ (c) $\sqrt{5gl}$ (d) $\sqrt{\frac{l}{g}}$

(11) The distance between the two rails is 1 m on a circular track of radius 400 m. The outer rail must be raised by how much distance so that the train moves with speed 72 km/hr without wear and tear

(a) 5 cm (b) 10 cm (c) 15 cm (d) 20 cm

(12) When a bucket filled with milk upto rim is rotated in vertical circle, the milk does not fall because

(a) milk has high adhesive force with bucket
(b) centrifugal force balances the force of gravity
(c) centre of gravity does not play any part
(d) force of gravitation does not act on it

(13) A particle of mass 100 g is tied to one end of string of length 1m. It rotates in a vertical circle. When the string makes an angle of 60^o with the vertical, its velocity is 2 m/s. The tension in the string in this position $\left(g = 9.8 m/s^2\right)$.

(a) 0.4 N (b) 0.09 N (c) 0.89 N (d) 0.98 N

(14) A small body of mass 0.1 kg swings in vertical circle, at the end of chord of length 1m. If speed is 2 m/s when chord makes angle of 30^o with vertical, find tension in the chord $\left(g = 9.8 m/s^2\right)$

(a) 0.4 N (b) 0.85 N (c) 0.98 N (d) 1.25 N

(15) A cyclist moves in circular track of radius 100 m. If the coefficient of friction is 0.2, then the maximum speed with which the cyclist can take a turn without leaning inwards in m/s is

(a) 9.8 (b) 1.4 (c) 14 (d) 1.0

(16) A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 rpm. Keeping the radius constant, the tension in the string is doubled. The new speed is nearly

(a) 14 rpm (b) 2.25 rpm (c) 10 rpm (d) 7 rpm

(17) A certain string breaks under 45 kg.wt. A mass of 0.1 kg is attached to this string of length 5 m and whirled in horizontal circle. The maximum number of revolutions per sec without breaking the string is :

(a) 4.72 rps (b) 47.2 rps (c) 472 rpm (d) 47.2 rpm

(18) A 2kg stone at the end of a string 1 m long is whirled in a vertical circle. The speed of the stone is 4 m/s. The tension in the string will be 52 N, when the stone is ($g = 10\ m/s^2$)

(a) at the bottom of the circle (b) at the top of the circle
(c) Half way down (d) none of these

(19) The total energy in the string revolving in a vertical circle with a mass 'm', radius 'r' and acceleration due to gravity g at the lowest point is

(a) $\frac{5}{2}$ mgr (b) $\frac{2}{3}$ mgr (c) 6 mgr (d) 3 mgr

(20) The angular displacement of a particle performing circular motion is, $\theta = \frac{t^4}{60} - \frac{t}{4}$. Where θ is in radians and t is in seconds. The angular acceleration of a particle at the end of 10 seconds is

(a) 10 rad/s² (b) 20 rad/s² (c) 30 rad/s² (d) 40 rad/s²

(21) A particle of mass m is moving in a circular path of constant radius 'r' such that its centripetal acceleration 'a' is varying with time 't', $a_c = k^2rt^2$ where 'k' is constant. The power delivered to the particle by the forces acting on it is

(a) Zero (b) mk^2r^2t (c) $\frac{mk^4r^2t^5}{3}$ (d) $\frac{mk^2r^2t^2}{3}$

(22) An object of mass 100 grams is whirled in a horizontal circle of radius 1 meter. If it performs 120 revolutions per minute, its angular velocity is

(a) 4π rad/s (b) 2π rad/s (c) π rad/s (d) $\pi/2$ rad/s

(23) Two bodies of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. They make one revolution in same time. Their angular speeds are in the ratio

(a) $m_1 : m_2$ (b) $m_1r_1 : m_2r_2$ (c) $\frac{m_1}{r_1} : \frac{m_2}{r_2}$ (d) 1 : 1

(24) An astronaut is rotating in a rotor of radius 4 m. If he can withstand acceleration upto 10 g, then the number of permissible revolution is,

(a) $\frac{5}{2\pi}$ rad/s (b) $\frac{2\pi}{5}$ rad/s (c) $\frac{10}{2\pi}$ rad/s (d) $\frac{5}{10\pi}$ rad/s

(25) The concept used in spin drier to dry clothes is

(a) centripetal force (b) gravitational force (c) centrifugal force (d) frictional force

EVALUATION PAPER - CIRCULAR MOTION ANSWER KEY

1 **b**	2 **b**	3 **a**	4 **a**	5 **b**	6 **a**	7 **b**	8 **b**	9 **a**	10 **b**
11 **b**	12 **b**	13 **c**	14 **d**	15 **c**	16 **d**	17 **a**	18 **a**	19 **a**	20 **b**
21 **b**	22 **a**	23 **d**	24 **a**	25 **c**					

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